

Hooke's law can be used for some Springs, only linear Springs.

We get 10 pts for quiz 1

othello.mech.northwestern.edu

A truck collides w/ a car. Which sustains greater damage?

The car.

~~X~~ - FILES

The TRUTH is out there

Laws of physics based on notion of energy, not force.

$W = T \Delta x$ Tension times Δ in distances
 $work = -G \Delta A$ Surface tension times Δ in area ?
 $W = P \Delta V$ Pressure times Δ in volume

$\frac{N}{m^2} \cdot m^3 = N \cdot m = J$ amt of work needed to move 1 kg mass 1 m.

You inc. the kinetic energy of the molecules of whatever is heated. C is 4.18 J for water.

8.338 is 2×4.18 , & is really important.

Universal Gas Constant

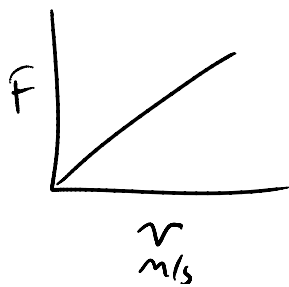
$\frac{1}{mc^2}$ of a photon = 8.338

mc^2 of a photon is amu

$\frac{1}{amu} = 6.02 \times 10^{23}$ Avogadro's #.

Nothing in the world touches anything else b/c of the repulsive force between the molecules.

The truck & car exert the same amt of force on each other



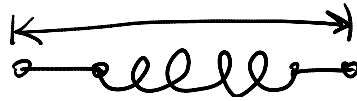
Linear

$$F = b \dot{v}$$

$$F = k x$$

parameters

$$(N) = (N/m) m$$



lumped parameters - you take the entire body as a whole

Through variable in the case of damper is force. Has same value throughout system.

Across variable in damper is velocity. Difference between 2 ends

Dampers

Relates F
to $\dot{v}$



Relative $\dot{v}$ of 2 ends is important

Sign convention (tension (+), compression (-))

2 ppl pull on off ends of ideal damper

Both of you pull w/ same force

The damper lengthens b/c there's tension forces.

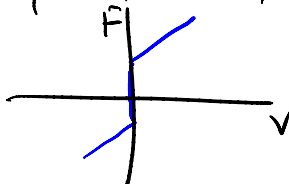
Left end of damper moving to the right at 10 cm/sec

Right end moving to the right at 5 cm/s

Damper's velocity is neg.



Graph of a "sticky" damper



You push & push but no motion.

In real life, all dampers are sticky

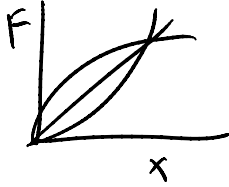
it's just a matter of what force & ...

begins moving at. An ant couldn't move the damped.

Hooke's Law

$$F = kx$$

but sometimes not linear.



$$F = -k \frac{\partial T}{\partial x} \quad \text{heat transfer}$$

thermal conductivity, could be $f(T)$

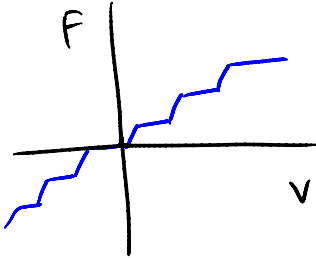
X-FILES

NUMB3RS

24

STAR TREK

If there's movement but no force:



Superconductor - the fluid moves without friction

Super fluid

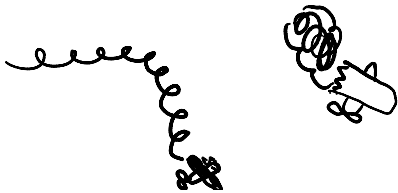
Super luminosity - laser photons that move & are all exactly same wavelength

ANY DAY NOW...
CLASS WILL END!

$$h\nu \leftarrow \text{frequency}$$

$$6.626 \times 10^{-34} \text{ J.s}$$

Coherent photons



If x is (-), F is (-)

Springs

Wednesday, March 28, 2007

7:06 AM

leaf springs  Coil   $\tau = k\theta$
 $t = \text{torque}$

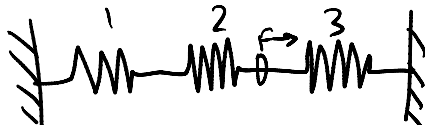
$$E = \int F dx = \int_{x_0}^x kx dx = \frac{1}{2} kx^2$$

$$E > 0, x > x_0$$

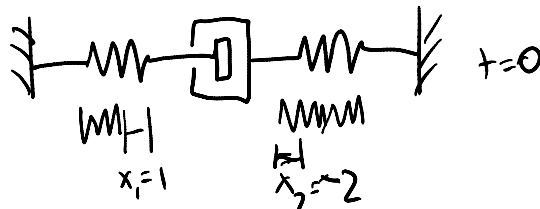
$$E < 0, x < x_0$$

If $x > x_0$, F is (+) tension

If $x < x_0$, F is (-) compression

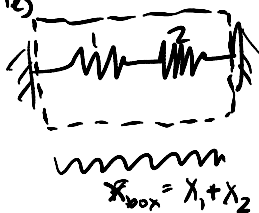


Forces are same in spring 1 & 2 but not 3



When 1st released at $t=0$, syst moves naturally to new position, & then continues to move slowly in 1 direction

Series



$$F_1 = k_1 x_1 \quad F_2 = k_2 x_2$$

$$f_1 = f_2 = f$$

$$x_1 + x_2 = \frac{f_1}{k_1} + \frac{f_2}{k_2} = f \left(\frac{1}{k_1} + \frac{1}{k_2} \right)$$

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Series here acts like || in electricity

0845 ... 5 more min !!!

mmmm
r=0.2m

60 in arm span



$$F = k(1.5 - 0.2 \text{ m})$$

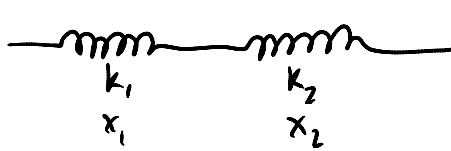
↑
estimate force

$$50 \text{ g} = k(1.5 - 0.2 \text{ m}) \quad 50 \text{ g is arbitrary}$$

$$500 \text{ N} = F$$

$$\frac{50}{1.5 \text{ m}} = k$$

$$a = \frac{dv}{dt} \quad \frac{v_2 - v_1}{t_2 - t_1} \rightarrow \text{cannot be zero}$$



$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_{eq} < k_1 \quad k_{eq} < k_2$$

$$x = x_1 + x_2$$

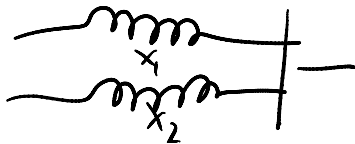
$$F = kx$$

$$F = -x$$

$$\frac{F}{k}$$

$$F_1 = F_2 = F$$

etc



$$x_1 = x_2 = x$$

$$F_{box} = F_1 + F_2$$

$$(k_1 + k_2)x = F_{box}$$

$$k_{eq} = k_1 + k_2$$

$$k_{eq} > k_1 \quad k_{eq} > k_2$$

$$F_x = \dot{w} = \dot{E}$$

$$F_v = \dot{w} = \dot{E}$$

$$\text{Watts}$$

$$\tau d\theta = w$$

$$\tau d\omega = \dot{w}$$

$$P dV = dW$$

$$P \dot{V} = \dot{E}$$

$$\text{Watt}$$

$$V \frac{dz}{dt} = \dot{E} \quad z \text{ is charge}$$

$$\frac{dz}{dt} = i, \text{ current in Amperes}$$

Force Balance

- Assume all forces positive, in tension

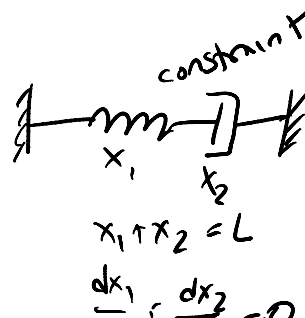
- FBD

- Apply Newton's 2nd Law

Geometric Continuity

- No gaps, interpenetrations (

- Series & Parallel rules



$$\frac{dx_1}{dt}, \frac{dx_2}{dt} = 0$$

$$V_1 \cdot V_2 = 0$$

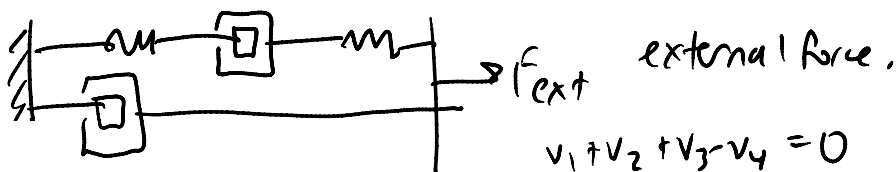
$$V_S \text{ r } V_d = 0$$



$$V_1 + V_2 + V_3 - V_4 = 0$$

$$F_1 = F_3 \quad F_2 = -F_4$$

$$F_3 = -F_4$$



$$v_1 + v_2 + v_3 - v_4 = 0$$

but $F_2 \neq -F_y$ b/c

Text #0

$\uparrow \quad \downarrow \quad \uparrow \quad \downarrow \quad \uparrow \quad \downarrow$

$$x_3 + x_4 = L, \text{ and } L \text{ is}$$

$\mathbb{N} - A$ can add to 0

$$\text{If } v_1 > 0, v_3 < 0$$

fixed so adds to 0

$$f_2 = v_2 b_2$$

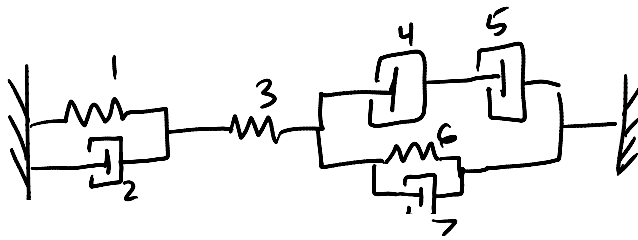
$$\text{so if } v_2 > 0, f_2 > 0 \quad x_1 > 0, v_2 > 0$$

$$F_1 = k_1 x_1$$

$$F_2 = F_1, \text{ so } x_1 > 0$$

Displacement of Damper

Monday, April 02, 2007
7:08 AM

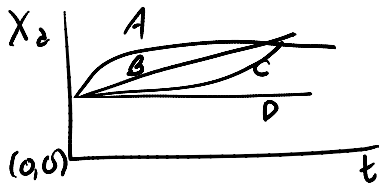


$$\begin{aligned} V_1 &= -V_3 - V_6 \\ f_4 + f_5 &\neq f_6 + f_7 \\ f_3 &\neq f_4 + f_6 \\ V_4 &= V_5 \\ V_1 &\neq V_2 \\ v_2 \neq 0, x_1 > 0, \text{ then } x_3 > 0 \end{aligned}$$

At $t=0$ damper's displacement $x_d = 1$

Which graph(s) should be soln to eqn of motion of this system if the damper is ideal?

What graph(s) could be soln to eqn of motion if damper is not ideal?



$$f_d = b v_d$$

$$v_d = \frac{dx_d}{dt}$$

v_d must be constant if f_d is constant

$$\int v_d dt = \int dx_d$$

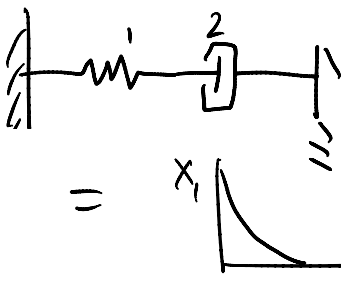
$$v_d t + x_{d0} = x_d$$

so (B) if ideal

if $f=0$, then (D) is sticky damper

A & C are nonideal b/c nonlinear

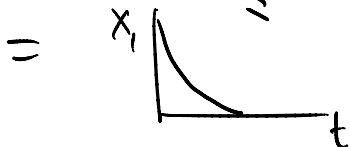
v_d being a variable, means you can't integrate dt.



$$x_1 + x_2 = L$$

$$V_1 + V_2 = 0$$

Geometric constraint



$$f_s = k x_s$$

$$f_d = b v_d$$

Constitutive laws

$$f_s = f_d$$

$$v_s + v_d = 0$$

$$v_s = -v_d = -\frac{f_d}{b} = -\frac{f_s}{b} = \sqrt{\frac{-k x_s}{b}} \quad \text{or} \quad \frac{dx_s}{dt}$$

$$v_s = v_d = -\frac{1}{b} = -\frac{1}{b} = \left[\frac{1}{b} \left(v_s = \frac{dx}{dt} \right) \right]$$

initial value problem

ODE

$$\frac{dx}{dt} = -Ax$$

$$\int \frac{dx}{x} = \int -A dt$$

$$\ln|x| = -At$$

$$x = K e^{-At}$$

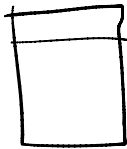
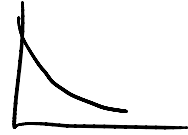
$$x = C e^{-\frac{k}{b}t}$$

I.C

$t=0$

$$x = x_0 = C$$

$$x = x_0 e^{-\frac{k}{b}t}$$



water flows out, graph looks like



Euler's Method

$$v(t) = -\frac{k}{b} x(t)$$

Loop

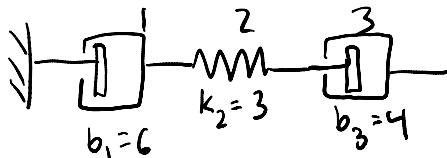
$$x(t+dt) = x(t) + v(t)dt$$

Plot

$$\frac{dy}{dx} = f(x, y)$$



Applied Numerical Method (w/ MATLAB)
Steven C. Chong



$$f_1 = b_1 v_1 \quad f_2 = k_2 x_2 \quad f_3 = b_3 v_3 \quad \text{Constitutive laws}$$

$$f_1 = f_2 = f_3 \quad \text{Force balance}$$

$$x_1 + x_2 + x_3 = L \quad \text{Geometric Continuity}$$

$$v_1 + v_2 + v_3 = 0 \quad (\text{take derivative})$$

$$v_2 = -v_1 - v_3$$

$$v_2 = -\left(\frac{f_1}{b_1} + \frac{f_3}{b_3}\right)$$

$$v_2 = -f \left(\frac{1}{b_1} + \frac{1}{b_3} \right) = -f \left(\frac{b_1 + b_3}{b_1 b_3} \right) = -\left(k_2 x_2 \right) \left(\frac{b_1 + b_3}{b_1 b_3} \right)$$

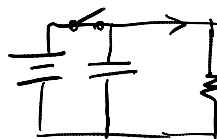
$$\frac{dx_2}{dt} = -\left(\frac{k_2}{\left(\frac{b_1 b_3}{b_1 + b_3} \right)} \right) x_2$$

$$\frac{dx_2}{dt} = -\frac{k}{b} x_2$$

$$b_{eq} = \frac{b_1 b_3}{b_1 + b_3}$$

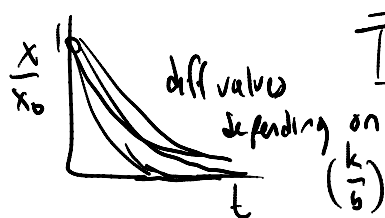


$$x = x_0 e^{-\left(\frac{k}{b}\right)t}$$



This has a similar eqn to x

$$x = x_0 e^{-\dots}$$



eqn to x

$$\frac{\partial v}{\partial t} + w \nabla v = v \nabla^2 v$$

$$\frac{\partial \Gamma}{\partial t} + w \nabla \Gamma = \kappa \nabla^2 \Gamma$$

Compute $F_2(t=0)$, $V_1(t=0)$, $V_3(t=0)$, $V_2(t=0)$, Approx $x_2(t=.1)$, compute $V_2(t=.1)$ / Approx $x_2(t=.2)$

$$\text{At } t=0, x_2=4 \text{ (given)}$$

$$V_2 = \left(-\frac{k_2}{\frac{k_1 b_3}{k_1 + b_3}} \right) x_2 = -\left(\frac{5}{4} \right) \underset{1}{x_2} = -5$$

$$V_1 + V_2 + V_3 = 0$$

$$V_2 = -(V_1 + V_3) = -(2+3) = -5$$

$$x_2 = x_2 + \Delta t V_2 \quad \text{Euler's Method}$$

$$y' = f(x, y)$$

$$\frac{dy}{dx} = f(x, y)$$

$$\frac{\Delta y}{\Delta x} = f$$

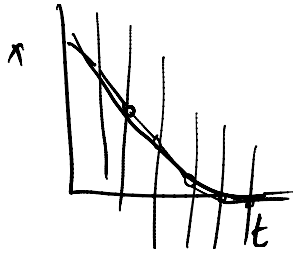
$$\frac{y_{i+1} - y_i}{\Delta x} = f$$

$$y_{i+1} = f(x, y) \Delta x + y_i$$

$$\begin{aligned} x_2(t=.1) &= x_1(t=0) + \Delta t V_2(t=0) \\ &= 4 + .1(-5) \\ &= 3.5 \text{ m} \end{aligned}$$

$$V_2(t=.1) = -\left(\frac{5}{4} \right) \underset{3.5}{x_2}(t=.1) = -4.375$$

x |||



Find displacement $x(t)$ for DEQ

Wednesday, April 04, 2007

7:10 AM

clear;

$k = 20$

$b = 10$

$\Delta t = .01$

$t(1) = 0$

$v = (-\frac{k}{b})^* x(1)$

for $i = 1:300$

$x(i+1) = x(i) + v * \Delta t$

$t(i+1) = t(i) + \Delta t$

end

plot(t, x)

It's missing $x(1) = \underline{\hspace{1cm}}$
Initial condition

V should be inside for loop or
it's $x(i)$ not $x(1)$

$300 * .01 = 3 \text{ s}$, so it
doesn't reach 10 sec

What is wrong?

Find soln $x(t)$ for DEQ

$$\frac{dx}{dt} = 2\sqrt{x}$$

$$\int \frac{1}{\sqrt{x}} dx = \int 2 dt$$

$$\sqrt{x} = t$$

$$x = t^2$$

$$x(t) = t^2$$



starts at $t=0$, $x_s = -2$
compression

A) $x_s(t) = 0$

B) $x_s(t) = -2e^{-3t}$

C) $x_s(t) = -2e^{3t}$

D) $x_s(t) = -\frac{2}{t+1}$

E) None

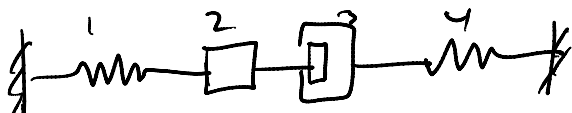
$x = C e^{-\frac{k}{b}t}$

Must be B b/c
it has (-) exp.

Masses

$F = ma$

$-f_1 + f_2 = m \frac{dv_2}{dt}$



a) $v_1 + v_2 + v_3 + v_4 = 0$

b) $f_1 = f_2 = f_3 = f_4$

X

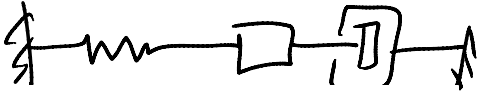
X

b/c $x_1 + x_3 + x_4 = L$

$v_1 + v_3 + v_4 = 0$

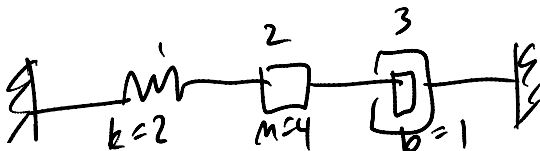
- c) $v_3 + v_4 = 0$ X
d) $m \frac{dv_2}{dt} = -f_1 + f_3 + f_4$ X
e) $f_1 = f_3$ X
f) $f_3 + f_4 = 0$ X

$-f_1 + f_3$ is all
goes through the mass
 $f_3 = f_4$



- a) if $x_1 < 0$, then $v_2 > 0$
b) if $\frac{dv_2}{dt} > 0$, then $x_1 < 0$
c) if $\frac{dv_2}{dt} > 0$, then $v_3 < 0$
d) if $x_1 > 0$, then $v_3 > 0$
e) if $v_2 = 0$, then $x_3 = 0$
f) if $v_2 = 0$, then $x_1 = 0$

X not necessarily
X $m \frac{dv_2}{dt} = -f_1 + f_3$ not necessarily
X same $k_1 x_1$ $b_3 v_3$
X
✓ X
X



Approximate $x_s(t=0.5)$

$$x_s = -2, v_m = 3 \text{ when } t = 0.4$$

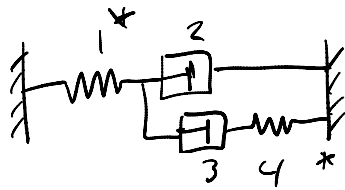
$$dt = 0.5 - 0.4 = 0.1 \text{ s}$$

$$v_{\text{mass}} = v_1 = v_s = \frac{dx_s}{dt}$$

$$x_s(i+1) = x_s(i) + v_s \cdot dt$$

$$x_s(i+1) = -2 + 0.3 = -1.7 \text{ m}$$

$$\dot{x}_s' = F(x_s, v_{\text{mass}}, \text{no velocity of damper})$$



$$f_1 = k_1 x_1 \quad f_2 = b_2 v_2 \quad f_3 = b_3 v_3 \\ f_4 = k_4 x_4$$

f_3 is not a state function
 $f_3 = f_4$



$$v_1 + v_2 = 0 \quad f_1 = f_2 + f_3$$

$$v_2 = v_3 + v_4 \\ v_1 + v_3 + v_4 = 0$$

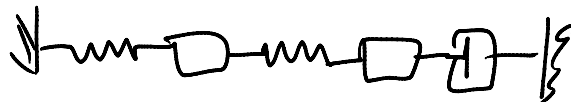
$$\dot{x}_1' = v_1 = -v_2 = -\frac{f_2}{b_2} = -\frac{f_1 + f_3}{b_2} \\ = \frac{-k_1 x_1 + k_4 x_4}{b_2}$$

$$v_4' = -v_2 - v_3 = \text{solve}$$

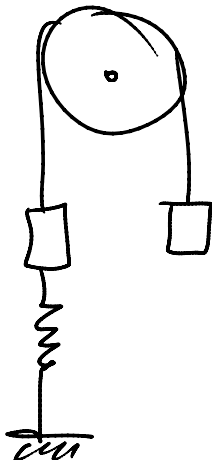
$$= \dot{x}_1' - v_3$$

How many state variables does this sys have?

(4)

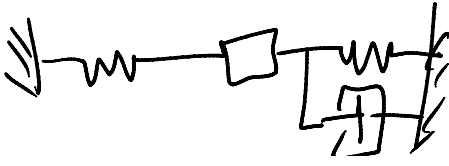


How many now?

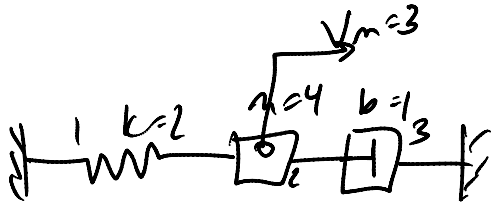


(2)

If you know one mass, you know the other



How many?
 (2)
 Once you know one
 x_5 you know the
 other.



$$t = .4$$

$$x_5 = -2 \quad v_m = 3$$

Approximate $x_5(.5)$
 Compute at $t = .4$ f_d & f_s , & a_m

Approx $v_m(.5)$

Is mass speeding up or slowing down? Why?

$$dt = .1$$

$$dx_1 = v_m$$

$$x_2 = x_1 + (v_m)dt$$

$$x(.5) = x(.4) + (3)(.1)$$

$$= -2 + .3 = -1.7 \text{ m}$$

$$f_3 = b_3 v_3 = -v_2$$

$$F_3 = -3 \quad f_3 = -4$$

$$f = kx$$

$$m \frac{dv_2}{dt} = m a_m$$

$$-f_1 + f_3 = m a_m$$

$$25 = m a_m$$

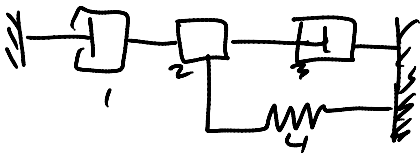
$$25 = \frac{dv}{dt}$$

$$25 dt = dv$$

$$3.025 = v$$

$$V = v + a dt$$

$$a > 0, \text{ so } V \uparrow$$



Ind. state variables?

$$x_4 \text{ or } v_2$$

(2)

$$v_1 + v_3 = 0$$

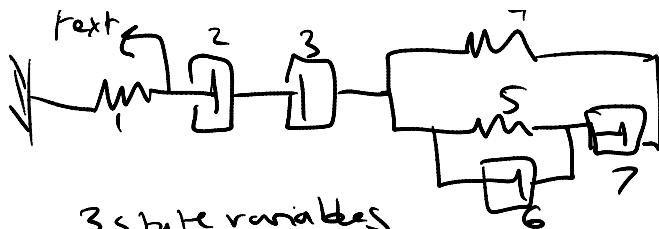
$$v_3 = v_4$$

$$m \frac{dv}{dt} = -f_1 + (f_3 + f_4)$$

$$\frac{dx_4}{dt} = v_4 = v_3 = \frac{f_3}{b_3}$$

$v_2 = v_3$?? X no, it's not

For x_2 x_4 x_1



3 state variables

$$v_1 + v_2 + v_3 + v_4 = v_{\text{end}}$$

Dear Journal:

$$v_4 = v_5 + v_7 \quad v_5 = v_6$$

Today, we learned what was on yesterday's quiz

Nonlinear: $f = k(x) \cdot x$

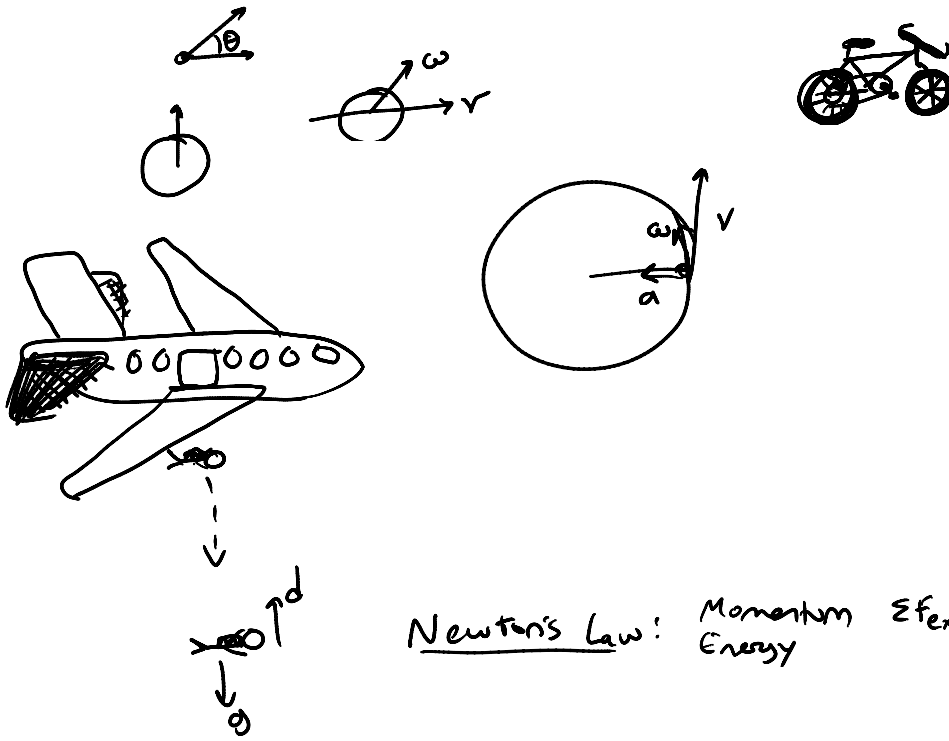
0841...
9 MORE MIN!

WHY CAN I
NOT LOSE 9
MIN LIKE
MULDER!?!

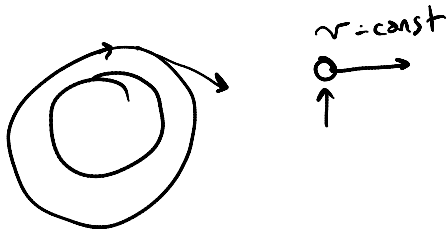
Trajectory, Acceleration

Monday, April 09, 2007

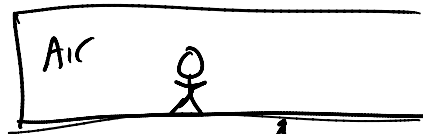
7:02 AM



Newton's Law: Momentum $\Sigma F_{ext} = ma = \frac{d}{dt}(mv)$
Energy

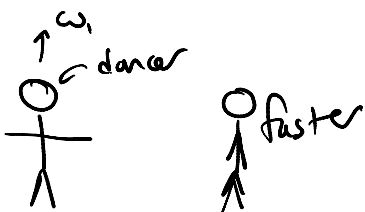


$r = \text{const}$



You could

- have a bag of apples & throw them, moving that way
- use a fan to propel yourself

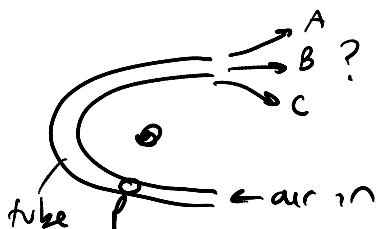


Energy is always constant.

Mass goes into energy

$$\Sigma F_{ext} = m_{tot} * a_{cm}$$

$$E = mc^2$$



trajectory of pellet when expelled
is B, straight

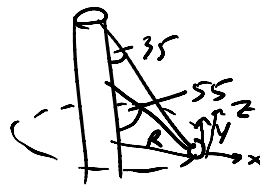
tube $\rightarrow$ air in

eventually, it will curve down. C. $\rightarrow$

At point P, V_{pellet} is ? The velocity in the x direction, V_x , is 0. Nonexistent. V_t , tangential, is pos.

Acceleration at P? 0. No V , constant V , so no a . Acceleration is in direction of the center, +x, +y in that position.

0840



Constant ω

$$a = r\omega^2 \checkmark a_x \text{ nonzero}$$

$$0 = \alpha = \frac{d\omega}{dt} \times a_y \text{ zero}$$

T contracts g $\times a_z \text{ zero}$

no V in x direct. $\times V_x \text{ zero}$

$v = r\omega$ tangent to curve $\checkmark V_y \text{ nonzero}$

$$\alpha = 0 \quad \times \alpha \text{ zero}$$

TANGENT STARTS

Here he goes...
appreciation of stuff...
not qualified to teach in
blah blah...

blah...
blah...

head hits desk

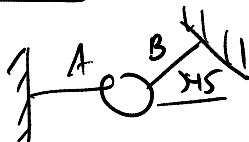
wha?? was someone talking?

AH! Thermo!! He's talking about
thermo again!

HELP! HELP! HELP! HELP! HELP! HELP!

TANGENT ENDS

0849



Someone cuts rope A

Immediately after cutting A, Tension in B is

a) zero

b) $mg \sin 45$

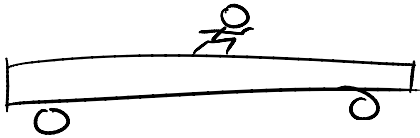
b/c no acceleration in y direction

c) more than $mg \sin 45$ b/c of centripetal acceleration

d) mg

b/c acceleration

d) my
e) can't solve



run from end to end.
30 kg plank 3 kg person
How far does CM person +
plank move up & back



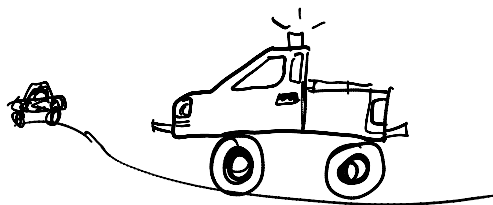
$$P = 3 \text{ kg}$$

$$\text{plank} = 30 \text{ kg}$$

The CM does not wobble for the whole system stays the same.

The plank moves so CM of system remains fixed

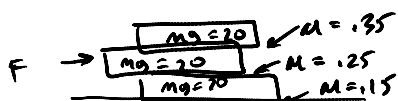
A truck starts from rest on yellow budge, drives to left. Force that made the truck move?



c) the force of friction acting to the right
 c) less than Mmg is size of force but it isn't having rubber

10 ton 18 wheel truck & 1 ton 4 wheel car going 60 mph.
 Both slam on brakes @ same moment. Both brakes lock
 all wheels coefficient of friction = .42

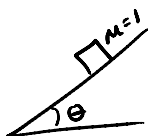
The stopping distance is the same.



Min force required to cause motion?

B) 916

Once motion begins, force of friction acting on yellow blocks less than ?



As $\theta \uparrow$ slowly from $0 - 89^\circ$, when does sliding happen?

$M = 1$, $f = N = mg \cos \theta$
 It will slide at $\theta = 45^\circ$.

I'm hungry

As $\theta \uparrow$, $0 \rightarrow 89^\circ$, when does tumbling happen?

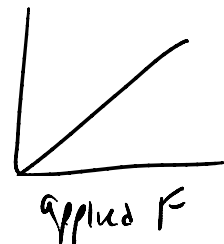
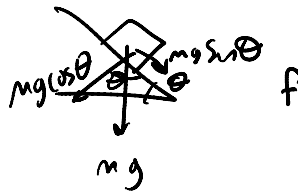
$$Mg \cos(\theta) = N = F_n$$

Wednesday, April 11, 2007
7:06 AM

$$Mg \cos \theta = N = F_n$$

$$f = \mu N$$

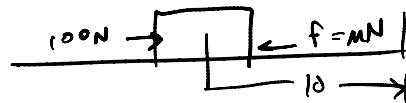
$$\mu = \tan \theta$$



$$F_x = mg \sin \theta - \mu_k N = a_x$$

$$m = 10 \text{ kg}$$

$$\mu_s = 0.5$$



$$\text{work? } v_2 \text{ @ } x = 10 \text{ m? } E_{\text{lost?}}$$

$$\Sigma W = (100 - 49) 10 = 510 \text{ J}$$

$$f = \mu mg$$

$$f = -49 \text{ N}$$

$$W_f = (-49) 10 = -490 \text{ J}$$

$$W_F = (100) 10 = 1000 \text{ J}$$

$$1000 \text{ J} - 490 \text{ J} = 510 \text{ J}$$

490 → Heat

$$W = \frac{1}{2} m v^2$$

$$10.1 \text{ m/s} = v$$

$$\mu_s = 0.5, \mu_k = 0.3 \quad a) \text{ brakes, onset of slip } \mu = \mu_s \quad b) \text{ brakes lock } \mu = \mu_k$$

$$v = 30 \text{ m/s. } x_{\text{stop?}}$$

$$-f = ma$$

$$-\mu mg = ma$$

$$-mg = a$$

$$-a g = -\frac{dv}{dt} \frac{dx}{dx}$$

$$\frac{d}{dx} \left(\frac{1}{2} v^2 \right) = a$$

$$v^2 - v_0^2 = 2 a \Delta x$$

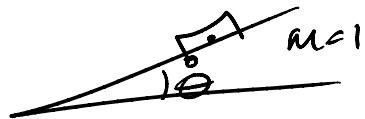
$$a) a = -\mu g = -(0.5)(9.81) = -4.9$$

$$\Delta x = 91.8 \text{ m}$$

$$b) a = -\mu g = -(0.3)(9.81) = -2.94$$

• - 2.1.1

The cube from yesterday tumbles at $\theta = 45^\circ$



When the moment is 0, it will tumble

$$F \frac{L}{2} = N \left(\frac{L}{2} - x \right)$$

I'm hungrier

Collisions

Friday, April 13, 2007
7:08 AM

2 shopping carts initially at rest. Frictionless, identical, except one holds giant watermelon so mass is $>$. Child in one cart throws watermelon to child in other cart. Carts move apart.



a. The catching cart (on left) is faster

b. The throwing cart (on right) is faster

c. Moving at same speed

$$m_w v_{w1} + m_1 v_1 = m_w v_{w2} + m_2 v_2$$

$$= 0$$

$$-m_2 v_2 = m_w v_{w2}$$

so

$$m_w v_w + m_1 v_1 = (m_w + m_1) v_1$$

$$v_1 = \left(\frac{m_w}{m_w + m_1} \right) v_w \leftarrow$$

$$\vec{v}_2 = -\frac{m_w v_{w2}}{m_2}$$

Hippo on a barge, each weigh 1 ton. Initially hippo is in middle of barge, length L . Hippo walks to front of barge & stops &...

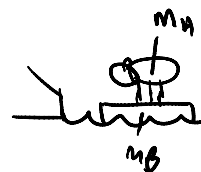
a) He can now walk onto shore

b) The barge is still moving toward shore

c) The barge is still moving away from shore

d) Barge is $L/2$ from shore & stationary

e) Barge is $L/4$ from shore & stationary



As  moves $\leftarrow$,

$$m_H = m_B$$

barge moves $\Rightarrow$.

Hippo runs off barge at 2 m/s.

a) Barge stops when hippo falls off

b) Barge stops when hippo splashes to a stop

c) Barge continues at 1 m/s

d) Barge continues at 2 m/s

e) This can't happen

f) The rhinoceros is nearsighted, not the hippo



f) The rhinoceros is nearsighted, not the hippo

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

$$KE_{initial} = KE_{final}$$

Conservation of
momentum

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

if $m_A = m_B$ the velocities are z.

$$\boxed{\frac{v_B' - v_A'}{v_A - v_B} = e}$$

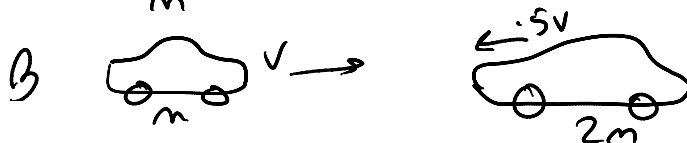
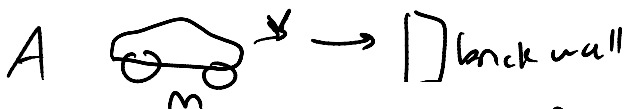
Coefficient of restitution

$$0 \leq e \leq 1 \quad \begin{cases} e=1 & v_{relative} = v_{relative}' \quad \text{perfectly elastic} \\ e=0 & v_{relative}' = 0 \quad \text{perfectly plastic} \end{cases}$$

Relative velocity goes away like they have the same speed

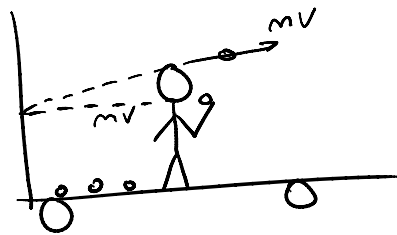
Dissipation - KE of larger scale moves to KE on small scale

Concept ?



All 3 collisions are totally inelastic which brings car on left to a halt?

All will



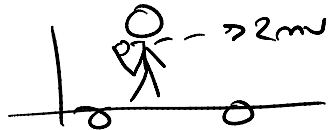
Balls bounce straight back

- a) moves to right
- b) moves to left
- c) doesn't move
- d) depends



$$mv + mv = 2mv$$

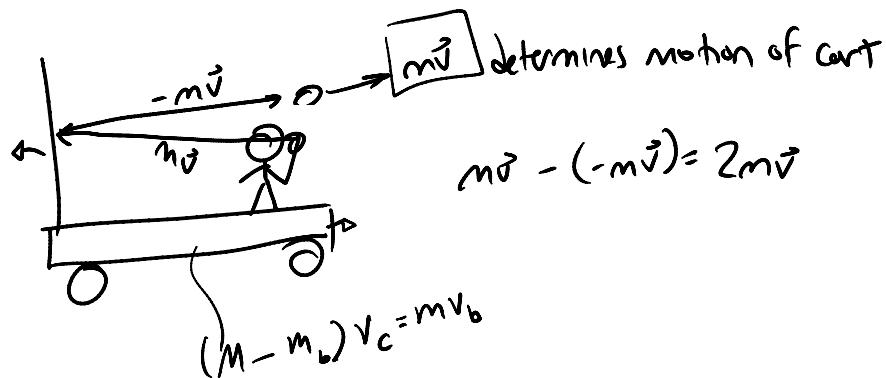
same thing as



$$(M_1 - m_b) \vec{v}$$

Conservation Forces

Monday, April 16, 2007
7:05 AM



Kickback from throwing is neutralized by ball hitting bar

Kinetic & Potential energy add to conserved energy. Total energy is fixed
Conservative Forces:

No energy lost, work path independent

$$\int \vec{F}_{\text{conserv}} \cdot d\vec{r} = -\Delta W$$

$$U = V_2 - V_1, \dots \text{ for conservative forces}$$

Conservation of E:

$$[PE + KE]_1 = [PE + KE]_2 + E_{\text{lost}}$$

$$V_{\text{grav}} = mgy$$

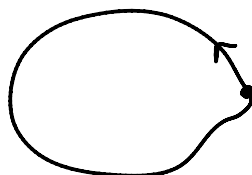
$$V_{\text{spring}} = \frac{1}{2} kx^2$$

$$W = \int \vec{F} \cdot d\vec{x} = \int m \frac{d\vec{v}}{dt} \cdot d\vec{x} = \int m d\vec{v} \cdot \frac{d\vec{x}}{dt} = \int_1^2 m v dv = \frac{1}{2} (v_2^2 - v_1^2)$$

Gravity is a conservative force

When friction present, a certain amt of work goes into heat

No change in energy in syst. when you do this:



Work is path independent

path function
not path dep.
 $\Delta = U + W$

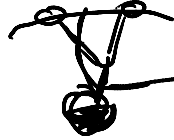
$$U = \frac{1}{2} kx^2 + \text{Const} \quad \text{for a spring} \quad \text{int. energy}$$

Conservative Forces

~~Squash ball hits wall~~

Indy 500

Golf shot off a tee



Positive Work

Tuesday, April 17, 2007
7:11 AM

Positive work:

Extend a spring that was relaxed by 20 cm, compress a dumper by 3 in
Ride a bike 10 mi.

After Tangent 33 min long:

"We don't have to go into this. And we won't, because it's not important for this course. I am telling you this so you remember for the rest of your life."

A bungee cord is properly adjusted over a tar pit. Somehow jumper overshoots & is in the tar. Explanation?

- (A) Jumper ate a large meal & didn't adjust bungee
- (B) Didn't just drop, but jumped upward
- (C) Jumped downward
- (D) Heat from tar pit increased k of cord
- (E) "decreased k of cord"
- (F) Attractive force due to stickiness of the tar

Drop a superball from 5 ft off Earth onto surface & it bounces back w/ height of 5 ft. w/ each bounce Earth recoils, gaining a bit of momentum toward sun. Does this eventually pound Earth into sun?

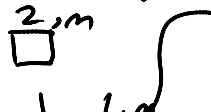
a $\times$ 600000 pulls Earth back up between bounces
CM is fixed.

We start on today's slides.

Ring 1 is on bottom half of a frictionless double U pipe while ring 2 falls through left end of pipe.

1. What would happen if collision of 2 rings is perfectly elastic?

A) Both move together



(B) Ring 2 stops, Ring 1 moves

E) Ring 2 moves back up to left. Ring 1 moves forward/pit to right

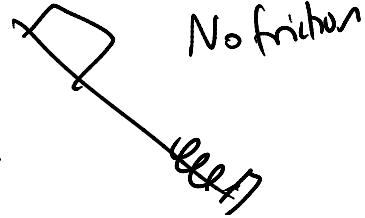
2. Can ring 2 push ring 1 out from right end of pipe?

(A) Yes, if it has enough PE to start w/

B) No

C) Ring 2 will drop out there too

Block released from rest on inclined plane slides down
compresses spring & rebounds back up.



~~A) KE is zero at moment it firsts contacts
ring~~

(B) PE of block is most (+) at moment of release

C) KE is largest when it first contacts spring

D) PE spring is >est at moment the rebounding block is
leaving contact w/ it

E) Greatest value of KE block at any time = greatest value
PE spring at any time

F) Block rebounds exactly to its initial height at release

Tractor beam is unilateral force, in which source A exerts a force on victim B, but victim B doesn't exert a force on A

- a) Not possible, violates N's 3rd law
- b) Momentum could not be conserved for A+B sys
- c) Conservation of energy couldn't hold for A+B sys
- d) tractor force could not be conservative force field for B
- e) interaction of A w/ B mediated by tractor force could not be elastic interaction
- f) No, the tractor beam is bilateral (b/c there have been ships that escape from it)



$$PE = mgh \quad (10\% \text{ loss}) \quad \therefore \frac{9}{10} h$$

$$\left(\frac{9}{10}\right)^n 4 = ?$$

$$[PE + KE]_1 = [PE + KE]_2 + E_{\text{lost}}$$

$$U = \int F dx \quad \text{Power} = \text{rate of work} = \frac{dU}{dt} = P = Fv$$

$$U = \int P dt$$

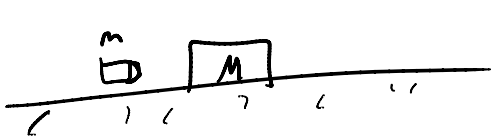
$$U = \int P dt$$

$$W = \int P dt \quad \downarrow W = P \Delta t$$

$$V = SP \Delta t$$

Power in Watts = J/s

Work in $J = Nm$



$$m \ll M$$

Initial velocity of bullet v
Coeff friction μ .

Where does box stop moving, distance D ?

When does box stop sliding, time T ?

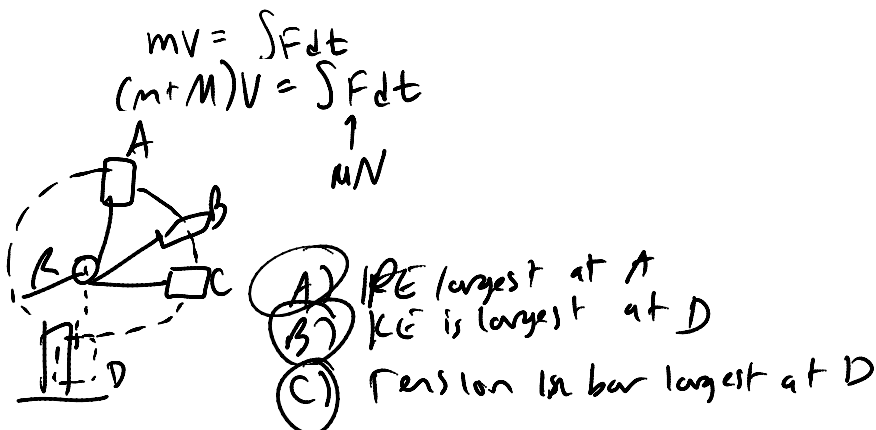
$$\int F dt = p = mv$$

$$mv = (m+M)v$$

$$KE = \frac{1}{2}(m+M)v^2 = f x \approx W$$

$$\frac{1}{2}(m+M)v^2 = \mu N x$$

$$\frac{(m+M)v^2}{2\mu N} = x$$



Tension in bar when hammer hits sample?

A) Mg B) $4Mg$ C) $5Mg$

Power of impact if hammer stops at D w/in Δt after hitting sample?

A) $2RMg/\Delta t$ B) $4RMg/\Delta t$ C) $5RMg/\Delta t$

$$a = \frac{v^2}{R}$$

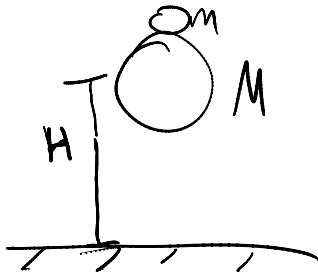
$$m \frac{v^2}{R} = ma$$

$$mgh = \frac{1}{2}mv^2$$

$$mg + 4Rmg = 5mg$$

$$v^2 = 4Rg$$

$$P = \frac{dE}{dt} = \frac{\frac{1}{2}mv^2}{dt} = \frac{\frac{1}{2}4Rg}{dt} = \frac{2Rg}{dt}$$



Can m go to qh?

$$(m+M)gh = \frac{1}{2}(m+M)v_0^2$$

$$mv_0 + Mv_0 = mV + mv$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}Mv_0^2 = \frac{1}{2}mV^2 + \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = mgh$$

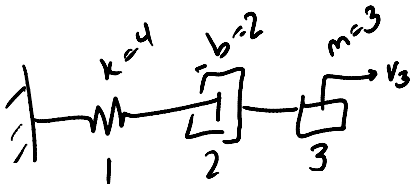
$$v_0 = \sqrt{2gh}$$

$$V = \frac{M\sqrt{2gh} + m\sqrt{2gh} + mv}{M}$$

$$v = \frac{-\sqrt{2gh}(-3M+m)}{m+M}$$

$$h = \frac{H(m-3M)^2}{(m+M)^2}$$

if $m \rightarrow 0$
 $h = 9H$



$$v_1 + v_2 = v_3$$

$$f_1 = f_2$$

$$x_1' = v_1 = v_3 - v_2 = v_3 - \frac{f_2}{b} = v_3 - \frac{f_1}{b} = \boxed{v_3 - \frac{k_1 x_1}{b}}$$

$$\boxed{x_1' = v_3 - \frac{k}{b} x_1}$$

Cannot be solved b/c it contains unknown v_3

$$\frac{dv_3}{dt} = v_3' = -\frac{f_2}{m_3}$$

$$\boxed{v_3' = -\frac{k_1 x_1}{m_3}}$$

$\boxed{} = \text{State eqn}$

Those are coupled b/c they contain each other. So solve simultaneously

Given certain times, you can find certain things.

$$x_1(t=0.5) = 3$$

$$v_3(t=0.5) = 2$$

$$x_1'(0.5) = v_3 - 6 = 2 - 6 = -4$$

$$v_3'(0.5) = -4$$

$$v_3(0.6) = v_3(0.5) + h(v_3'(0.5)) = 2 + 0.1(-4) = 1.6$$

$$x_1(0.6) = x_1(0.5) + h(x_1'(0.5)) = 3 + 0.1(-4) = 2.6$$

$$dt =$$

$$k = 4$$

$$b = 2$$

$$m = 3$$

$$x_1(1) =$$

$$v_3(1) =$$

$$t(1) =$$

$$\text{for } i = 1:100$$

$$x_1p(i) = v_3(i) - 2x_1(i)$$

$$v_3p(i) = -(k/m) * x_1(i)$$

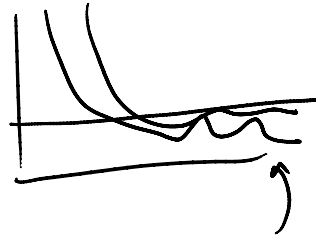
no acceleration

$$x_1(i+1) = x_1(i) + x_1p(i) * dt$$

$$v_3(i+1) = v_3(i) + v_3p(i) * dt$$

$$t(i+1) = t(i) + dt$$

end

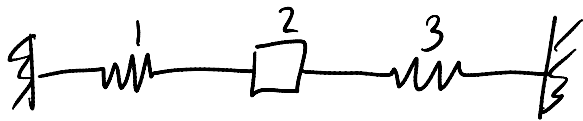


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State Vectors

Monday, April 23, 2007
7:05 AM



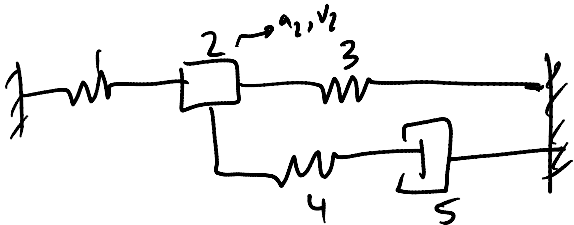
$\{v_2, x_3\}$ state vector
Find relationship useful
in eliminating x_1 .

- A) $x_1 = x_3$
- B) $v_1 + v_3 = 0$
- C) $x_1 + x_3 = 0$
- D) $v_1 + v_3 = \text{constant}$
- E) $x_1 + x_3 = \text{constant}$
- F) $x_1 = x_2$

$$x_1 + x_3 = L$$

$$v_1 + v_3 = 0$$

$$\frac{dx_1}{dt} + \frac{dx_3}{dt} = 0$$



$x_1 = x_3$
b/c of S, 1 is not
dependent on 4 or vice
versa

State variables:

$$x_1, v_2, x_4$$

Parameters:

$$k_1, m, k_2, k_3, b_5$$

Dynamic Variables:

$$f_2, v_1, x_5,$$

OMG moment brought to you by:



Geometric continuity eqns? :

$$v_1 \neq v_2$$

$$v_1 + v_3 = 0 \text{ or } v_3 = -v_1$$

$$v_1 + v_4 + v_5 = 0 \text{ or } v_4 + v_5 = -v_1$$

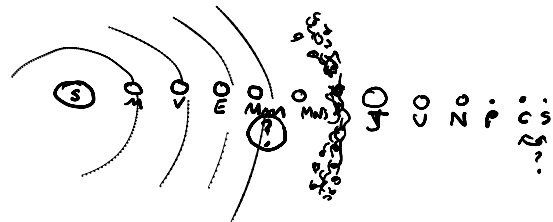
$$\frac{dx_1}{dt} + \frac{dx_4}{dt} + \frac{dx_5}{dt} = 0$$

$$x_1 + x_4 + x_5 = C = L$$

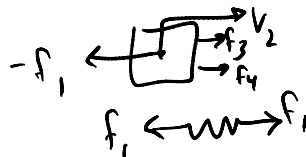
v_1 is redundant w/ v_3 , reduce set of state
variables to

$$\{x_1, v_2, x_4\}$$

"Traffic rules have nothing
to do with the physical
situation even though the
situation is... physical.
Anyway..."



Force balance eqns



$$m \cdot a = v_1' = -f_1 + f_2 + f_3$$

$$m_2 - m_2 v_2 \quad k_1 x_1 \quad -k_3 x_1 \quad k_4 x_4$$

$$v_2' = \left(\frac{k_1 + k_2}{m_2} \right) + \frac{k_4}{m_2} x_4$$

$$x_4' = v_4 = -v_1 - v_5$$

$$= -v_2$$

$$v_5 = \frac{f_5}{b_5}$$

$$-F_1 + F_3 + F_4 = m_2 a_2$$

$$F_4 = F_5$$

$$F_1 = k_1 x_1$$

$$F_2 = m_2 a_2$$

$$\text{etc}$$

$$F_4 = F_5$$

state eqns look like:

$$x_1' = \text{function of } (x_1, v_2, x_4)$$

$$v_2' = f(x_1, v_2, x_4)$$

$$x_4' = f(x_1, v_2, x_4)$$

$$x_1 = v_2$$

$$v_2' = a_2 = \frac{1}{m_2} [-F_1 + F_3 + F_4]$$

$$= \frac{1}{m_2} [-k_1 x_1 + k_3 x_3 + k_4 x_4]$$

$$= \frac{1}{m_2} [-k_1 x_1 - k_3 x_1 + k_4 x_4]$$

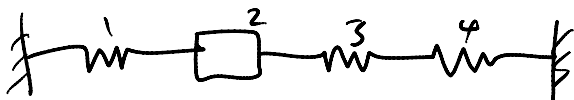
state vector

$$x_4' = -v_2 - \frac{k_4}{b_5} x_4$$

$$\begin{Bmatrix} x_1 \\ v_2 \\ x_4 \end{Bmatrix} = \bar{x}$$

$$\begin{Bmatrix} x_1' \\ v_2' \\ x_4' \end{Bmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -k_1 - k_3/m_2 & 0 & k_4/m_2 \\ 0 & -1 & -k_4/b_5 \end{pmatrix} \begin{Bmatrix} x_1 \\ v_2 \\ x_4 \end{Bmatrix}$$

$$[x'] = [A] * [x]$$



Find. state variables?

$$\textcircled{2} \{v_2, x_3\} \text{ or some other combo}$$

What will eliminate x_4 ?

$$k_4 x_4 = k_3 x_3$$

Superposition, using MATLAB to solve physics

Tuesday, April 24, 2007
7:11 AM

$$\left. \begin{array}{l} k_1 = \\ k_2 = \\ k_4 = \\ m_2 = \end{array} \right\} \text{ give parameters}$$

$$t(1) = _$$

$$t(n+1)$$

$$n =$$

$$\Delta t = \frac{t(n+1) - t(1)}{n}$$

$$\left\{ \begin{array}{l} x(1) \\ v_2(1) \\ x_4(1) \end{array} \right\} \text{ give stuff}$$

for (i = 1:n)

$$x_1 p(i) = 0; x_1(i) + v_2(i) + \overset{?}{0} x_4(i)$$

$$v_2 p(i) = -\left(\frac{k_1 + k_2}{m}\right) x(i) = 0 v_2(i) + \frac{k_4}{b_2} x_4(i)$$

$$x_1(i+1) = x_1(i) + x_1 p(i) \Delta t$$

$$v_2(i+1) = v_2(i) + v_2 p(i) \Delta t$$

$$x_4(i+1) = x_4(i) + x_4 p(i) \Delta t$$

$$t(i+1) = t(i) + \Delta t$$

end

plot(t, x1, v2, x4 etc)

$$(x(:, 1)) = [3, 2, 1]' = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -\left(\frac{k_1 + k_2}{m}\right) & 0 & \frac{k_4}{b_2} \\ 0 & -1 & -\frac{k_4}{k_5} \end{bmatrix}$$

for i = 1:n

$$xp(:, i) = A * x(:, i)$$

$$x(:, i+1) = x(:, i) + xp(:, i) \Delta t$$

$$t(i+1) = t(i) + \Delta t$$

end

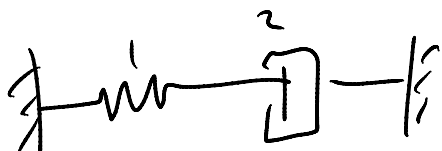
FIRST CONTACT



SECOND CONTACT

$v_{m1}(t) = 3 \sin 4t$, $v_{m2}(t) = -6 \sin 4t$ is a soln. So
 $v_{m1}(t) = 6 \sin 4t$, $v_{m2}(t) = 12 \sin 4t$ is also a soln.

If $v_{m1}(t) = 3 \sin 4t$, $v_{m2}(t) = -6 \sin 4t$ is a soln to l.n. DEQ,
 then $v_{m1}(t) = 3 \sin 8t$, $v_{m2}(t) = -6 \sin 8t$ is not a soln.



$$V_1 + V_2 = 0$$

$$f_1 = f_2$$

$$x_1' = V_1$$

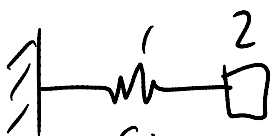
$$= -\frac{V_2}{\frac{k}{b_2}}$$

$$= -\frac{f_1}{\frac{k}{b_2}}$$

$$= -\frac{k}{\frac{k}{b_2}} x_1$$

$$x_1(t) = e^{-\frac{k}{b_2} t}$$

$$\text{then } x_1'(t) = (-\frac{k}{b_2}) e^{-\frac{k}{b_2} t}$$



State eqns

$$x_1' = V_2$$

$$V_2' = -\frac{k}{m} x_1$$

$$x = \begin{Bmatrix} x_1 \\ V_2 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1' \\ V_2' \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ V_2 \end{Bmatrix}$$

$$x_1'' + \frac{k}{m} x_1 = 0$$

$$x_1(t=0) = ?$$

$$x = A e^{\pm i \sqrt{\frac{k}{m}} t}$$

$$x = A \left[\cos \sqrt{\frac{k}{m}} t \pm i \sin \sqrt{\frac{k}{m}} t \right]$$

$$x = A_1 \cos \sqrt{\frac{k}{m}} t + A_2 \sin \sqrt{\frac{k}{m}} t$$

Euler Formula, Oscillations

Wednesday, April 25, 2007

7:10 AM

Euler Formula

$$e^{jy} = \cos y + j \sin y \quad j = \sqrt{-1}$$

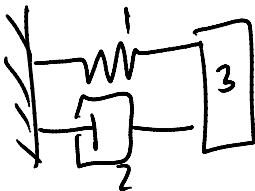
$$e^{-jy} = \cos y - j \sin y$$

$$x_s'' + \frac{k}{m} x_s = 0 \rightarrow x_s = A e^{rt}$$

$$x_s = A e^{\pm j \sqrt{\frac{k}{m}} t}$$

$$x_s = A \left[\cos \sqrt{\frac{k}{m}} t \pm j \sin \sqrt{\frac{k}{m}} t \right]$$

$$x_s = A_1 \cos \sqrt{\frac{k}{m}} t + A_2 \sin \sqrt{\frac{k}{m}} t$$



Oscillations decay

Motion decays w/ no oscillations

only doesn't move if zero ΣC_j , stiffer damper

Oscillates forever only if $b=0$

$$x_1' = v_3$$

$$v_3' = -(k_1 x_1 + b_2 v_3) / m$$

$$v_3'' + (b v_3' + k v_3) / m = 0$$

$$x = (e^{rt}) \quad x'' + \frac{b}{m} x' + \left(\frac{k}{m}\right) x = 0$$

$$r^2 + \frac{b}{m} r + \frac{k}{m} = 0$$

$$r = \frac{-\frac{b}{m} \pm \sqrt{\left(\frac{b}{m}\right)^2 - 4\left(\frac{k}{m}\right)}}{2}$$

If b is non zero, our soln will decay

If $\sqrt{\text{thing}}$ is $(-)$, diff than if $\sqrt{\text{thing}}$ $(+)$

$$x = C e^{(-1 \pm j)t}$$

$$\text{if } m=1, k=3, b=2$$

$$b/c \quad r = -1 \pm 5\sqrt{-4}$$

$$r = -1 \pm 5(2j)$$

$$r = -1 \pm j$$

$$x'' + \frac{b}{m} x' + \left(\frac{k}{m}\right) x = 0$$

$$x'' + 2x' + 2x = 0$$

$\frac{1}{2}$ ton chain hoist stuff. Has multi-speed or 1 speed.

Good velocity source
Ind. of force.



test machine for materials.

What is best if you want to test at a const. ^{displacement} rate?

Screw-driven b/c it's exact

What is best if you want to test at constant load?

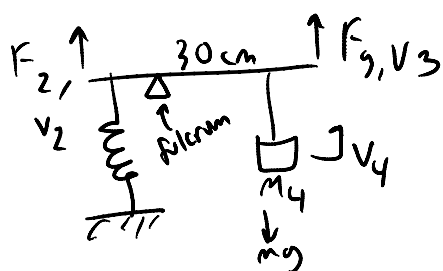
Hydraulic driven

Springs like rubber bands are not good $\dot{v}$ or $\dot{F}$ sources.

Rocket engine is a pretty good force source

Ground during an earthquake is a pretty good velocity source.

A diagram of a beam with a vertical support at the right end. Two downward forces, F_1 and F_2 , are applied to the beam. A counter-clockwise curved arrow indicates a moment. The reaction force at the support is labeled R .



1 & 2
 $F_1 \neq F_2$
 $F_1 = -F_2$ ✓
 $V_1 = V_2$ ✓
 $V_1 \neq -V_2$

$$\begin{aligned} 2 & \neq 3 \\ F_2 & \neq F_3 \\ F_2 & = 3F_3 \checkmark \\ F_2 & \neq -3F_3 \\ v_2 & \neq 3v_3 \\ v_2 & \neq -3v_3 \\ 3v_2 & = -v_3 \checkmark \end{aligned}$$

$$\begin{aligned} & 3 \text{ & } 4 \\ & F_3 \neq F_4 \\ & F_3 + m_3 g \neq m_4 a_4 \\ & -F_3 + m_4 a_4 \\ & -F_3 - m_3 g = m_4 a_4 \quad \checkmark \end{aligned}$$

Find state eqn

$$\begin{aligned} F_1 &= -F_2 \\ -F_3 - mg &= ma_4 \\ F_2 &= 3F_3 \\ V_1 &= V_2 \\ 3V_2 &= -V_3 \\ V_3 &= V_4 \end{aligned}$$

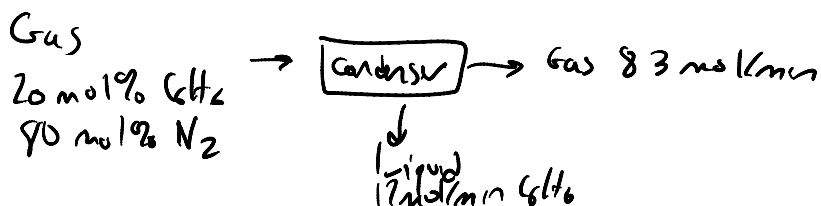
State var's: x_1, v_4

$$x_1' = -\frac{1}{3}v_4$$

$$v_4' = \frac{k_1 x_1}{3m} - g$$

2nd order form $x_1'' = -\frac{1}{3}v_4' \rightarrow -\frac{k}{9m}x_1 + \frac{g}{3}$

$$v_4'' = -\frac{k}{9m} \text{ something he didn't let us see}$$



How many balances can you write?

②

2 components here, but one is inert.
So N_2 is inert, no chem rxn. Mass
 N_2 & C_6H_6 is conserved

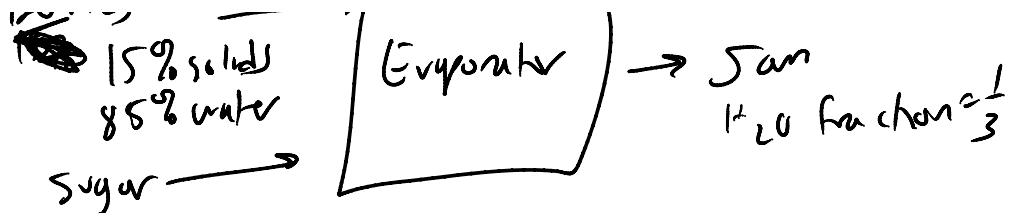
Mole fraction of gas stream is N_2 ?

$$X = \frac{n_i}{\sum n} \quad n = \# \text{ moles}$$

$$= \frac{80}{83.4} = \boxed{.96}$$

bc $80 + 3$, lost 17 mol C_6H_6





Strawberries & sugar mixed in 45:55 ratio. Mixture heated to evap water until residue contains $\frac{1}{3}$ water by mass. How many eqns can we write & how many unknowns do we know?

3 eqns 3 unknowns

How many lbs strawberries needed to make 1 lb jam?

$$\frac{M_{\text{strawberry}}}{M_{\text{sugar}}} = \frac{45}{55}$$

$$1 \text{ lb jam} = 1 - \frac{1}{3} = \frac{2}{3} \text{ solid}$$

$$\left(\frac{2}{3} \text{ lb}\right)_{\text{solid}} = .15 M_s + M_s \left(\frac{55}{45}\right)$$

$\uparrow$ mass of sugar

$$.5 = M_s$$

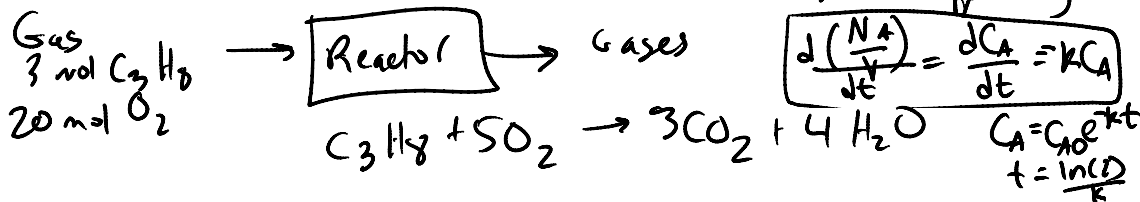
Gases

Monday, April 30, 2007
7:06 AM

rate of generation = - rate of consumption

$$M^{-1} h^{-1} m^{-1}$$

-rate = k Concentration²
↑
Thing is disappearing



Combustion of propane 90% is consumed (reacts w/ O₂). What is flow rate of exit stream?
.01 left of C₃H₈

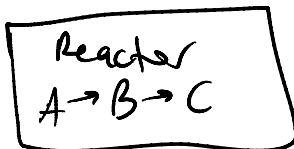
$$5 - 5(.9) = 5 - 4.5 = .5 O_2 \text{ left}$$

$$3(.9) = 2.7 CO_2$$

$$4(.9) = 3.6 H_2O$$

$$3(.01) = .3 CO_2 \text{ from } C_3H_8$$

$$3 \text{ mol } C_3H_8, 6.5 \text{ mol } O_2, 8.1 \text{ mol } CO_2, 10.8 \text{ mol } H_2O$$



Filled w/ A, then rxn started by raising temp. Write rxns for A, B, & C

$$r_A = -k_A C_A$$

$$r_B = k_A C_A - k_B C_B$$

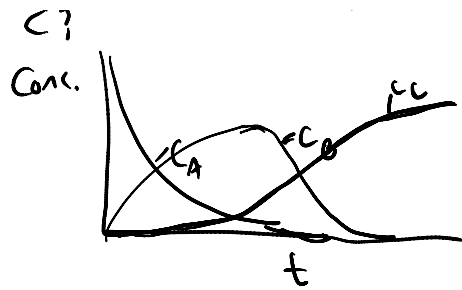
$$r_C = k_B C_B$$



If you plotted conc. of each species in the reactor as function of time, how would curves appear?

If B is valuable product how would you max. production of B?

If C is valuable product, " " " " " " "



$A \rightarrow 0$, all goes to B
 B depends on A & C, rate of decomposition.

$$F_A|_x - F_A|_{x+dx} + r_A A_c dx = 0$$

A_c = cross sectional Area

F_A = molar flux of A ($\frac{\text{mole}}{\text{time}}$)

$$\frac{dF_A}{dx} + r_A A_c = 0$$

Fick's Law

$$F_A = -D_A \frac{dC_A}{dx} A_c$$

$$-D_A \frac{d^2 C_A}{dx^2} + r_A = 0$$

Steady-state diffusion

Fick's Law $q_i'' = D \nabla p_i \rightarrow \frac{M}{V}$

$$\frac{M}{w_i V} = \frac{N_i}{V} = n_i = \text{molar concentration}$$

$$N^{\circ} k = R^{\circ} \text{ gas constant}$$

Convection transport is "forced" b/c the molecules do not have a choice

Fick's Law

$$n_i V_i = -D \nabla n_i \quad \text{concentration}$$

$$V_i = -D \frac{\nabla n_i}{n_i} = -D \nabla \ln(n_i)$$

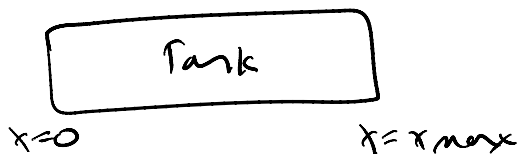
$$V_{\text{heat}} = -\alpha \nabla \ln T$$

$$V = \gamma \nabla \ln v$$

$$\tau = p V_i = -\mu \nabla v$$

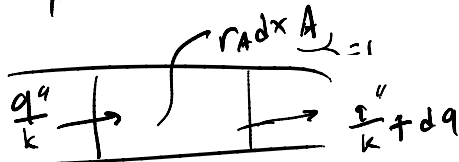
Stress

There is usually diffusion & convection.



Species A maintained at a C_{high} at $x=0$ or is at C_{low} at $x=x_{\max}$

What eqn & boundary condition describes this situation?



$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} + \dot{m}_{\text{generation}} = \left(\frac{dM}{dt} \right)_{\text{grad}}$$

If no generation, if $m_{\text{in}} > m_{\text{out}}$, $\frac{dM}{dt} \uparrow$.

Assume steady. No time.

$$\dot{m}_{in} - \dot{m}_{out} + \dot{m}_{generation} = 0$$

$$\frac{q}{x} - \left(\frac{q}{x} + dq \right) + r_A dx = 0$$

$$- \frac{dq}{dx} + r_A = 0$$

$$\frac{q}{A} = -D \nabla C_A$$

$$- \left(\frac{d}{dx} \left[-D \frac{dC_A}{dx} \right] + r_A \right) = 0$$

$$\boxed{+D \frac{d^2 C_A}{dx^2} + r_A = 0}$$

$r_A = 0$ if no reaction

$$\frac{d^2 C_A}{dx^2} = 0$$

$$x=L \quad \frac{dC_A}{dx} = 0$$

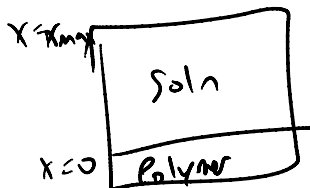
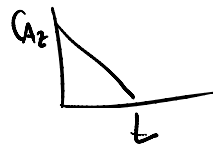
$$x=0 \quad C_A = C_{A0}$$

Unsteady state diffusion

$$\frac{d^2 C_A}{dx^2} = \frac{1}{D} \frac{dC}{dt}$$

$$C_A = A x + B$$

$$C_A = C_{A0} x + 0$$



Polymer containing a drug placed in an aq soln. Drug is in high conc. the polymer is infinitely soluble in soln. The soln is well stirred & release from polymer occurs by 1st order process.

Eqs + boundary conditions?

$$\frac{dC_s}{dt} = -kC_s$$

Balance on Soln $\frac{dC_s}{dt} = kC_p$

Balance on polymer $\frac{dC_p}{dt} = kC_p$

If it's in a stagnant soln,

Mass balance on polymer $\frac{dC_p}{dt} = k C_p$
 (balance on soln conc. $= \frac{dC_s}{dt} = D \frac{d^2 C_s}{dx^2}$)

For soln, need boundary (2) & initial condition (1)

1) $x=0, D \frac{dC_s}{dx} = k C_p$

2) $x=x_{max}, C_s=0$ or $\frac{dC_s}{dx}=0$

3) $t=0, C_s=0$

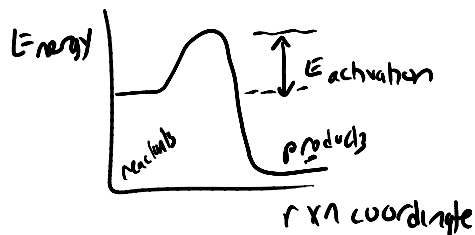
$r_A = -k C_A$

$\frac{dC_s}{dt} = k C_A$

$\frac{\partial p_i}{\partial t} + W_i \nabla p_i = D \nabla^2 p_i + \Omega_i$
 temporal convection diff

You can solve this analytically or numerically — activation energy

$k = \text{rxn rate const.}$ k written as $A e^{-E/kT}$



↑
frequency factor of collision rate

$\dot{w} = A (\pi C v') e^{-E/kT}$

rxn rate $\dot{w} = A e^{-E/kT} \cdot C_A^{v_A'} \cdot C_B^{v_B'}$

State functions are path independent

$$\oint f dx = 0 \quad \text{when} \quad \bigcirc \rightarrow$$

Voltage is an across variable
Current is a through variable.

The *Kirchoff Voltage Law (KVL)*, states the sum of the voltages added up around any closed loop is zero. This follows from the definition of voltage given above. We will revisit this rule again in the Loops and Junctions section.
Pasted from <http://othello.mech.northwestern.edu/ea3/book/elec1_2007/Volt.htm>

The (+) is upstream for everything but batteries

$$Q \quad [A][T]$$

amps time

$$V \quad [M][A]^{-1}[L]^2[T]^{-3}$$

mass amps length time

$$I \quad [A]$$

amps

$$E \quad [M][A]^{-1}[L][T]$$

electric field intensity

$$D \quad [A][L]^{-2}[T]$$

e- field density

$$R \quad [M][A]^{-2}[L]^2[T]$$

$$\sigma \quad [M]^{-1}[A]^{-2}[L]^{-3}[T]^3$$

conductivity

$$C \quad [M]^{-1}[A]^2[L]^{-2}[T]^4$$

Capacitance

Dumper is a resistor

Resistor develops relationship between V & I

Capacitor develops relationship between V & Q

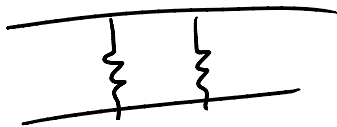


Flow variables: velocity, fluid flow, current
 Effort variables: force, pressure, voltage

Which quantities persist through - & - through an elmt?

- A) Force through a spring
- B) Velocity across a damper
- C) Force across a mass $\rightarrow$ Across or through is same thing
- D) Flow through a pump
- E) Voltage across a light bulb
- F) Current through a battery

$$\text{Amp} = \frac{\text{Coulomb}}{\text{sec}}$$



V is the same, parallel
 $R_{eq} = \sum_k R$



V is added, series.
 $\frac{1}{R_{eq}} = \sum_k \frac{1}{R}$

$$R = \frac{\rho}{A} L \quad \text{Resistance} = \frac{\text{resistivity}}{\text{Area}} \cdot \text{length}$$

Kirchhoff $\sum V = 0$

Current flow $\sum i_{in} = \sum i_{out}$

Current in = current out

Charge is conserved

$$P = IV$$

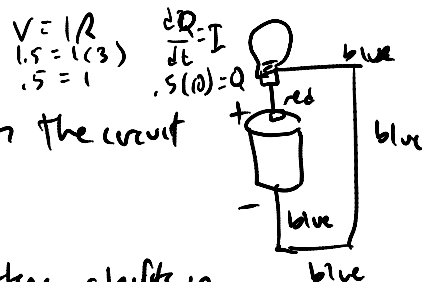
1.5 volt battery connected to 3Ω bulb for 10 sec

a) 0.5 A of current flows

b) The total charge that moves through the circuit in 10 s is 5 Coulombs

c) e^- move $\uparrow$ in red wire

d) Balance of + & - charge in battery shifts in



- favor of +
- e) Balance of + & - charge in batt shifts in favor of -
- f) no current flows b/c the circuit is not ground

$$\text{Work} = Vq = \text{Volts} \cdot \text{Coulombs} = \text{J}$$

$$P = \frac{V^2}{R} = I^2 R \quad \text{Watts}$$

$$P = V^2/R = i^2 * R \propto \omega \epsilon \lambda \lambda$$

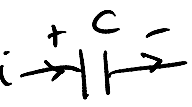
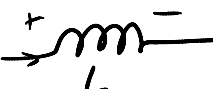
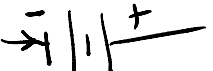
Pasted from <http://othello.mech.northwestern.edu/ea3/book/elec2_2007/Resistors.htm>

↖ x-mas lights
 All exp. the same current

Resistance, Capacitance, Inductance

Friday, May 04, 2007

7:01 AM

Elms Resistance		$V_R = iR$	$R = \frac{\rho L}{A}$ $\rho = \text{resistivity}$
Capacitance		$V_C = \frac{q}{C}$	$= \frac{L}{\sigma A}$ $\sigma = \text{conductivity}$
Inductance		$V_L = L \frac{di}{dt}$	
Battery		V_B	

C Capacitance Farad MF, nF, pF
(E-6) (E-9) (E-12)

L Inductance Henry

$$R = \frac{L}{\sigma A}$$

$$J = \sigma E \quad \text{electric field intensity}$$

$$E = -\nabla V \quad \leftarrow \text{grad voltage}$$

$$\begin{aligned} \vec{J} &= -\sigma \nabla V \\ i &= JA = -\sigma (\nabla V) A \\ \text{current} \quad \text{current density} \\ &= -\sigma A \frac{V_2 - V_1}{x_2 - x_1} \\ &= \frac{\sigma A}{L} (V_1 - V_2) \end{aligned}$$

$$\vec{i} = JA = \frac{\sigma A}{L} (V_1 - V_2)$$

$$\text{charge flow rate} \quad \frac{L}{\sigma A} = \frac{(V_1 - V_2)}{i} = R_{\text{electrical}}$$

... is related to

Electrical resistance is related to Ohm's Law.

Conduction of heat

Fourier's Law of Heat Conduction

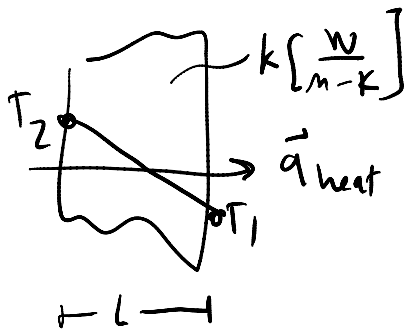
$$q'' = -k \nabla T$$

$$q = q'' A = -k A \left(\frac{T_2 - T_1}{x_2 - x_1} \right) \quad \mathcal{E} = -\sigma \nabla V$$

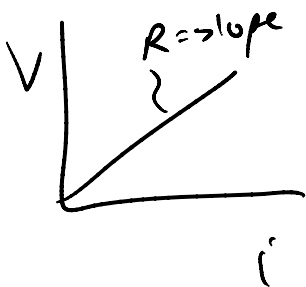
$$q = q'' A = \frac{kA}{L} (T_1 - T_2)$$

$$\frac{T_1 - T_2}{q} = \frac{L}{kA} = R_{\text{thermal}}$$

↑
heat flow rate



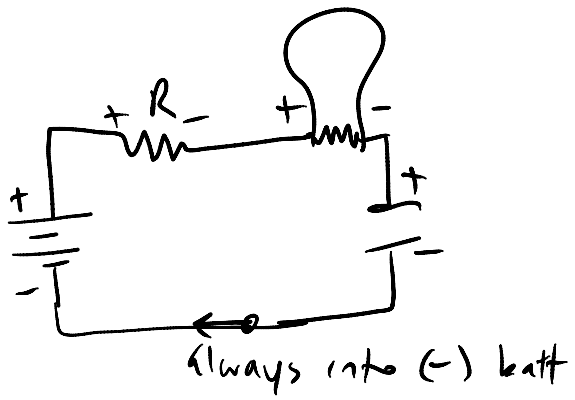
Flux induced by moving charge



$$q = -kA \nabla T$$

$$i = -\sigma A \nabla V$$

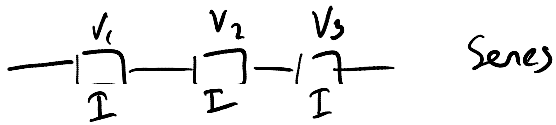
Effort / Force	Mech F	Electrical V	Thermal T
Flow	x, v, a	$\frac{dq}{dt} = i$	q ↑ heat flow rate



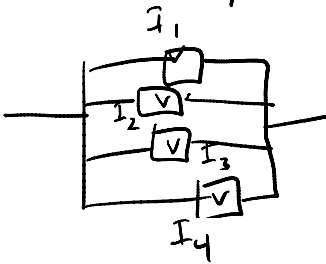
Balance of charge inside the battery remains fixed

Capacitance, Viscosity, Voltages

Monday, May 07, 2007
7:06 AM



$$V_T = \sum V_i$$



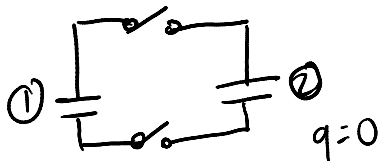
$$I_T = \sum I_i \quad \text{Parallel}$$

Capacitance

$$\frac{1}{\frac{1}{C}}$$

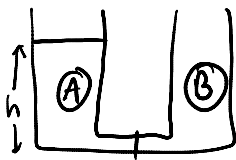
$$V_C = \frac{Q}{C} = \frac{1}{C} = \frac{\text{coulombs}}{\text{farad}} = \text{Volts}$$

$$\boxed{\begin{aligned} E &= \int V dq \\ &= \int \frac{q}{C} dq = \frac{1}{2} \frac{q^2}{C} \end{aligned}}$$



$$E = \frac{q^2}{2C}$$

$$E_2 = \left(\frac{q}{2}\right)^2 \frac{1}{2C} = \frac{q^2}{4C} \quad ???$$



initially, tank A has

$$PE = \frac{mgh}{2}$$

$$\text{open valve, } A = \frac{h}{2}$$

$$PE_2 = \frac{m}{2} g \frac{h}{2} + \frac{m}{2} g \frac{h}{2} = \frac{mgh}{4}$$

$\frac{1}{2}$ of PE is lost

Viscous dissipation
created when you
open the valve $\frac{E}{2}$
goes to "heat"

the center of mass moves.

But this is unresolvable

So

$$PE = \frac{m}{2} g \frac{h}{4} + \frac{m}{2} g \frac{h}{4} = \frac{mgh}{2} \quad \checkmark \quad \text{woo!}$$

$$\frac{m}{2} (g \frac{h}{4} + g \frac{h}{4})$$

$$\frac{m}{2} (g \frac{h}{2}) = \frac{mgh}{2} \quad ?$$

$$\frac{m}{2} (g \frac{h}{2} + g \frac{h}{2})$$

$$\frac{m}{2} (gh) = \frac{mgh}{2}$$

Sum of voltages around a loop = 0

Series: $i = i_1 = i_2 = \dots = i_n$ $V = V_1 + V_2 + V_3 + \dots + V_n = V_a - V_b$

Parallel: $i = i_1 + i_2 + \dots + i_n$ $V = V_1 = V_2 = V_3 = \dots = V_n = V_a - V_b$

FBI
 Federal Bureau of Investigation

$$\frac{+}{-} \quad \frac{-}{+} \quad \boxed{V_C = \frac{q}{C}}$$

$V_R = iR$

$$\boxed{V_C + V_R = 0}$$

$$\frac{dq}{dt} = q'_C - i_C = i_R = \frac{V_R}{R} = -\frac{V_C}{R} = -\frac{q_C}{RC}$$

$$\boxed{\frac{q'_C}{C} = -\frac{1}{RC} \frac{q}{C}}$$

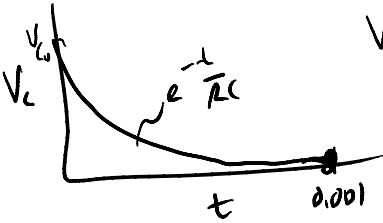
$$\frac{dq'}{dt} = i'_c = -\frac{1}{RC} i_c$$

$$V'_c = -\frac{1}{RC} V_c$$

$$d \ln V_c = -\frac{1}{RC} dt$$

$$V_c = A e^{-\frac{t}{RC}} \quad t=0, V_c = V_{c0}$$

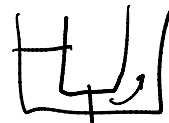
$$V_c = V_{c0} e^{-\frac{t}{RC}}$$



Resistance, Voltage, Work

Tuesday, May 08, 2007
7:06 AM

$$\begin{aligned}
 W &= \int P dV \\
 W &= E = \int P_{av} dV \quad V \text{ is Vol.} \\
 &= P_{avg} \Delta V \\
 &= \frac{mgh}{4} \quad (\text{the work that pushes this up})
 \end{aligned}$$

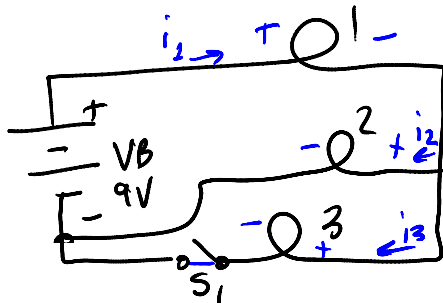


If you remove 1 bulb in a series set (X-mas tree lights) the others go out.

If you remove 1 bulb from a parallel set, it doesn't matter.

If you connect a single bulb to 120V, it will flash brightly & burn out

If you wanted to make a shorted series set w/ only 20 bulbs, you should not use 11.5 V. Too many V.



L_1 & L_2 are lit
w/ S_1 open
w/ S_1 closed,

S_1 open

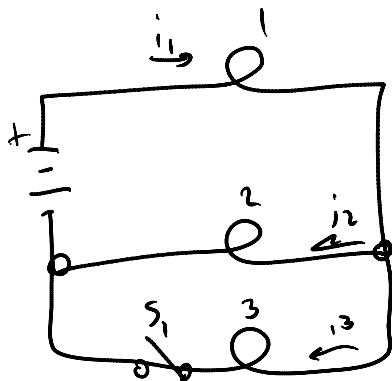
$$\begin{aligned}
 V_b &= V_1 + V_2 \\
 i_b &= i_1 = i_2 \\
 i_1 &= \frac{V_b}{R_1 + R_2}
 \end{aligned}$$

S_1 closed

$$\begin{aligned}
 V_3 &= V_2 \\
 V_1 &= V_2 + V_3 \\
 i_1 &= \frac{V_1}{R}
 \end{aligned}$$

$$i_1 = i_2 + i_3$$

$$V_1 = V_2 + V_3$$



$$V_A = V_2 = V_3$$

$$R_1 = R_2 = R_3 = R$$

S_1 closed

$$V_A = V_1 + V_2$$

$$V_2 = V_3$$

$$i_1 = i_2 + i_3$$

$$V_1 = i_1 R$$

$$V_2 = i_2 R$$

$$V_3 = i_3 R$$

$$i_1 R_1 + i_2 R_2 = 0$$

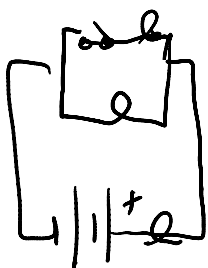
$$i = \frac{R}{5} \quad V_2 = 3 \quad V_3 = 3 \quad i_3 = \frac{2}{5}$$

L_1 brightens

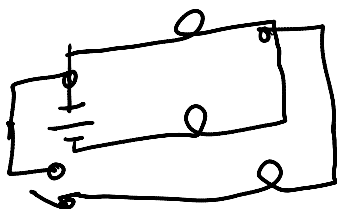
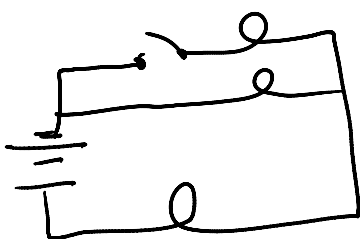
L_2 dim

L_3 brighten

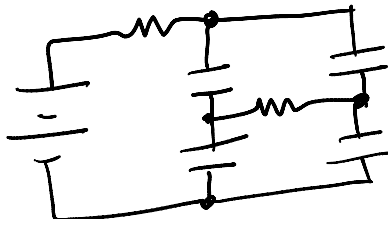
$$V = \frac{q}{C} \quad V' = \frac{1}{C} \leftarrow \text{That make sense} \quad \frac{dq}{db} = 1$$



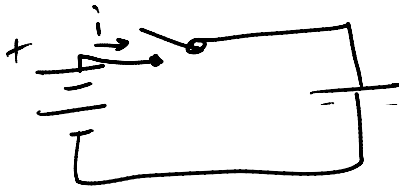
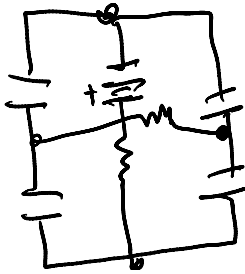
what is equivalent to this?



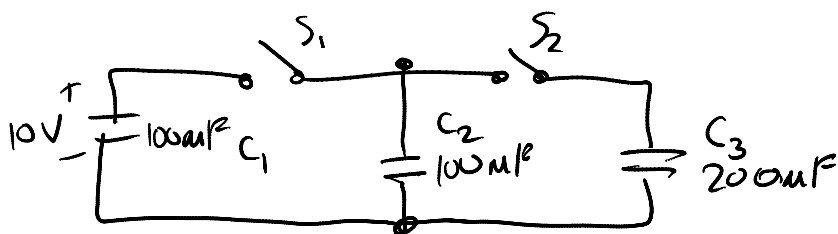
same thing ✓



which is equivalent?



What is steady-state current: ~~a~~ long after switch is closed? $\odot$



After S_1 closed, V 's across C_1 & C_2 ?

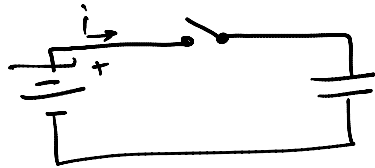
$$C_1 = 5V$$

$$C_2 = 5V$$

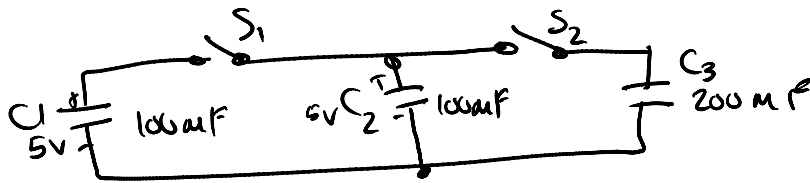
Circuits

Wednesday, May 09, 2007
7:04 AM

Time constant for RC circuit is RC



Steady state current long after switch is closed is zero



After S_2 is closed, what are V_s across C_2 & C_3 ?

$$V = \frac{Q}{C}$$

$$Q = CV = Q_2 + Q_3 \\ = V_{2\text{fin}} C_2 + V_{3\text{fin}} C_3$$

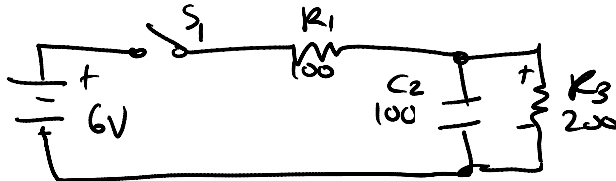
$$V_{2\text{fin}} (C_2 + C_3)$$

$\downarrow$ $\downarrow$
 100 200

$$C_2 = 2.5 \text{ V}$$

$$C_3 = 2.5 \text{ V}$$

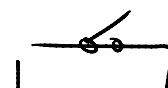
After S_1 closed in steady state, what is V across R_3 ?

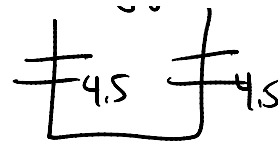
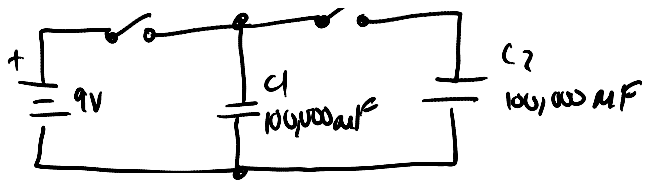


$$\frac{V_{\text{batt}}}{R_1 + R_2} = i$$

$$V_3 = R_3 i \\ V_3 = 4 \text{ V}$$

0826 Halfway Done!





Close S_1

$$V_{C1} = 9V$$

$$q_{C1} = 9C$$

Open S_1 , close S_2

$$V_{C1} = V_{C2} = 4.5V$$

$$q_{C1} = q_{C2} = .45C$$

Charge is conserved: $\frac{1}{2} q_{C1}$ flows from C_1 to C_2

Close S_1 :

$$V_{C1} = 9V \quad q_{C1} = 9C$$

$$E_{C1} = \frac{1}{2} CV^2 = 4.05J$$

Open S_1 , close S_2

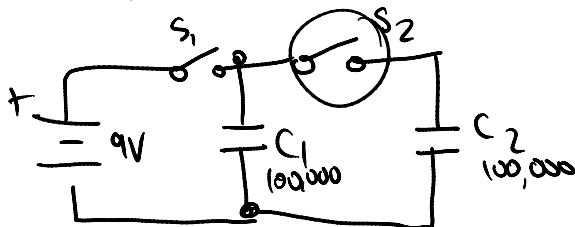
$$V_{C1} = V_{C2} = 4.5V$$

$$q_{C1} = q_{C2} = .45C$$

$$E_{C1} = E_{C2} = \frac{1}{2} CV^2 = 1.0125J$$

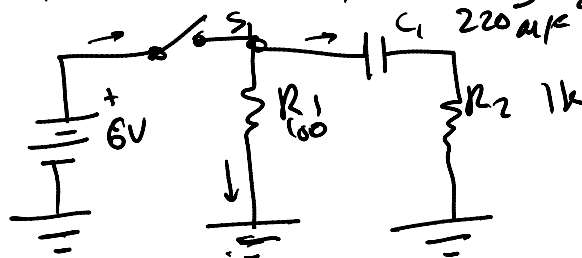
He's saying something... and it's totally lost on me.

Where is the rest?



It goes to S_2

When S_1 is 1st closed, what is zero?



$$V_{C1} = 0$$

$$V_{S1} = 0$$

$v > 1$ ~

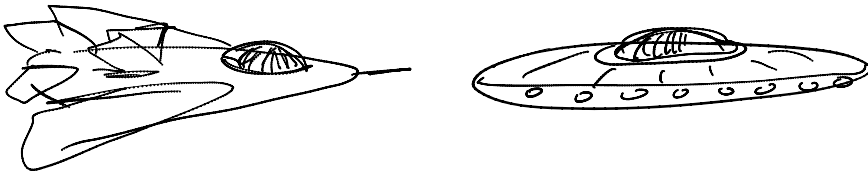
and V_{R2} ?? We don't know, since he disagrees w/ the slides.

What is time constant for C_1 ?

$$V_c = V_b(1 - e^{-t/\tau_c})$$

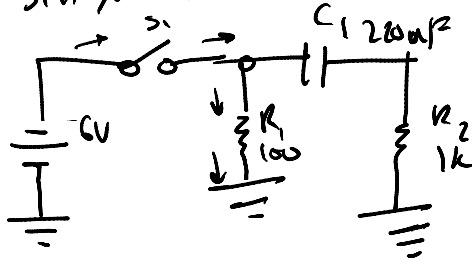
0842 ! 8 more min !

HOW HAVE I SURVIVED THIS LONG???



The answer is 122 sec

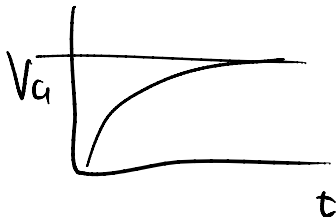
In steady state, which one zero?



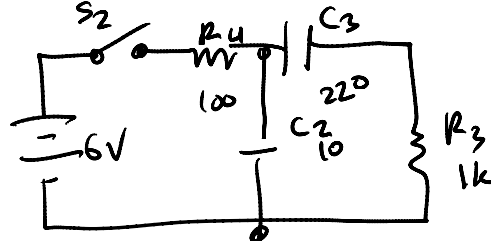
V_{S1} i_{C1}
 V_{R2}

What is the qualitative slope of charging of C_2 ?

which is the question for the circuit

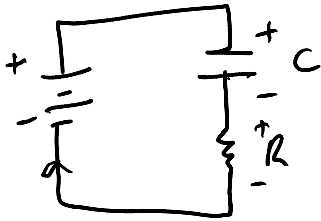


When S_2 is first closed, which are zero?



V_{C2} V_{R3} i_{C3}

In steady state, which are zero?
 i_{R4} , i_{C2} , i_{C3} , V_{R4} , V_{R3}



$$-V_B + V_C + V_R = 0$$

$$i_C = i_R = i$$

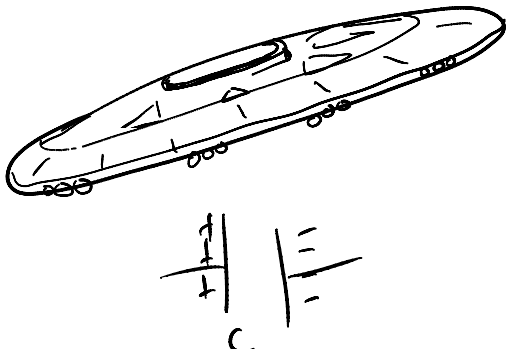
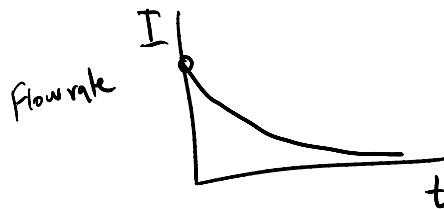
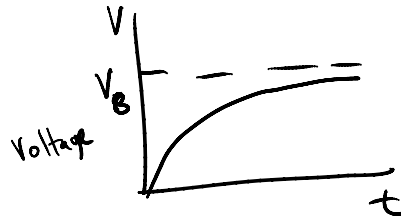
$$V_C = \frac{q}{C}$$

$$V_R = iR$$

$$V_C' = \frac{q'}{C} = \frac{i_C}{C} = \frac{i_R}{C} = -\frac{V_C}{RC}$$

$\left(\frac{V_B}{RC} \right)$
 source

$$V_C = V_B(1 - e^{-\frac{t}{RC}})$$



Mechanical systems:

elmts in parallel share velocity

elmts in series share force

$$\text{Flow} = \dot{v} \quad \text{Effort} = F$$

Electrical systems:

elmts in parallel share voltage

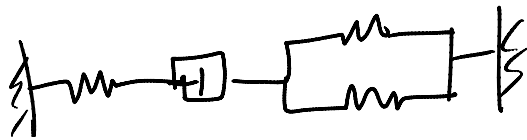
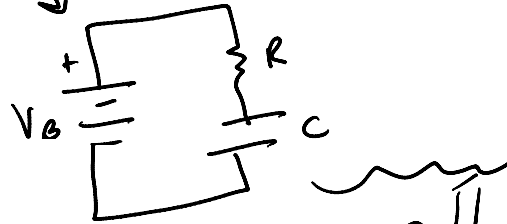
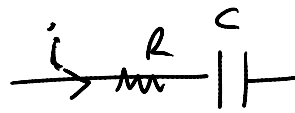
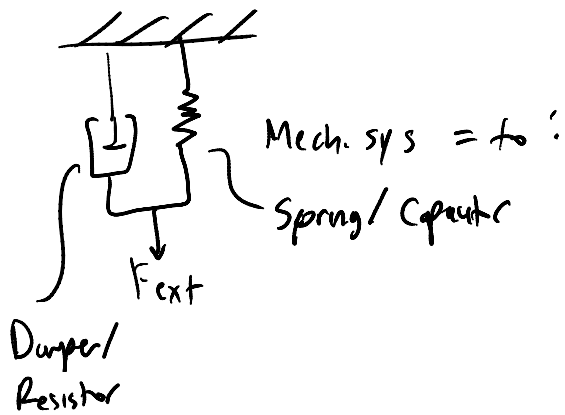
elmts in series share current

$$\text{Flow} = \text{current} \quad \text{Effort} = \text{Voltage}$$

For equivalence between systems, we want flows & efforts to be related in the same way

Resistors destroy energy & convert it to heat.

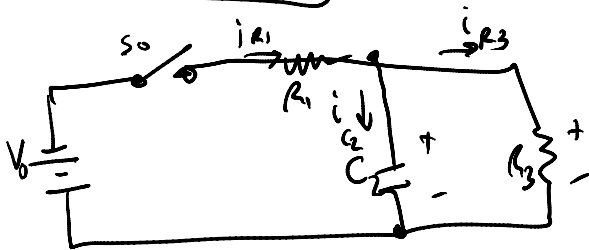
Force	Velocity	$\vec{v} A = \Phi$	Flow rate, Φ
Voltage	Current	$i = AS$	Flux thing
more like pressure than force			



Mech sys = to:



Volt-METER



when S_0 1st closed,

$$\begin{aligned} V_{C2} &= 0 \\ V_{R3} &= 0 \\ i_{R3} &= 0 \end{aligned}$$

In steady state,

$$i_{C2} = 0$$

state variables?

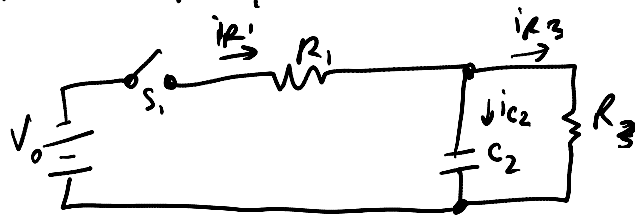
$$V_{C2} \text{ (or } q_{C2})$$

Find ind. loop eqns

$$V_0 + V_R + V_{C2} = 0$$

$$V_{R3} - V_{C2} = 0$$

Independent loop eqns



$$-V_0 = V_{R1} + V_{C2}$$

$$V_{C2} = V_{R3}$$

Junction eqns $i_{R1} = i_{C2} + i_{R3}$

Constitutive laws

$$V_{R1} = i_{R1} R_1$$

$$V_{R3} = i_{R3} R_3$$

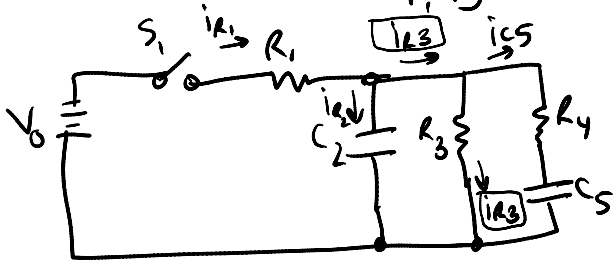
$$V_{C2} = Q_{C2} / C_2$$

$$Q' = i$$

$$V_{C2}' = \frac{\left(\frac{-V_0 - V_{C2}}{R_1} - \frac{V_{C2}}{R_3} \right)}{C_2}$$

Steady state, $V_{C2}' = 0$, solve for V_{C2}

$$V_{C2} = -V_0 \frac{R_3}{R_1 + R_3}$$

State Var. V_{C2}, V_{C5}

$$-V_0 + V_{R1} + V_{C2} = 0$$

$$i_{R1} = i_{C2} + i_{C5} + i_{R3}$$

$$-V_{C2} + V_{R3} = 0$$

$$-V_{R3} + V_{R4} + V_{C5} = 0$$

$$V_{C2}' = \text{something } V_{C2}, V_{C5}$$

$V_{C2}' = \text{something } V_{C2}, V_{C5}$
 $V_{C5}' = \text{something } V_{C2}, V_{C5}$
 do this one 1st

$$V_{C2}' = \frac{q_2}{C_2} = \frac{i_{C2}}{C_2}$$

$$V_{C5}' = \frac{V_{C2} - V_{C5}}{R_4 C_5}$$

$$V_{C5}' = \frac{\frac{V_{C2}}{R_3} - V_{C5}}{R_4 C_5}$$

$$V_{C5}' = \frac{(V_{C2} - V_{C5})}{R_4 C_5} \text{ b/c we don't know what he's doing}$$

$$V_{C2}' = \frac{1}{C_2} \left(\frac{-V_{C2}' - V_{C2}'}{R_1} - \frac{V_{C2}'}{R_3} - \frac{V_{C2} - V_{C5}}{R_4} \right)$$

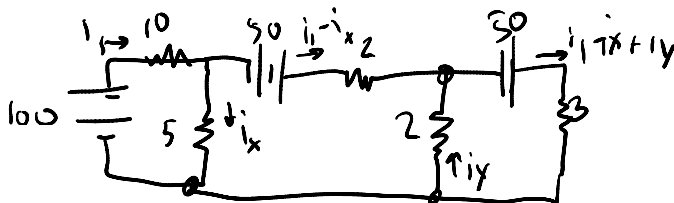
$$V_{C5}' = \frac{V_{C2} - V_{C5}}{R_4 C_5}$$

S: Agent Mulder, My name is Dana Scully. I've been assigned to work with you.

M: Oh, isn't it nice to suddenly be so highly regarded. Who did you tick off to get this assignment, Scully?

S: Actually, I'm looking forward to working with you. I've heard a lot about you.

M: Oh, really? I was under the impression that you were sent to spy on me.



$$100 = 10i_1 + 5i_x$$

$$50 = 2(i_1 - i_x) - 2i_y - 5i_x$$

$$50 = 3(i_1 - i_x + i_y) + 2i_y$$

$$i_x = -3.88A$$

$$i_y = 1.51A$$

Inductor, Capacitor

Tuesday, May 15, 2007

7:04 AM

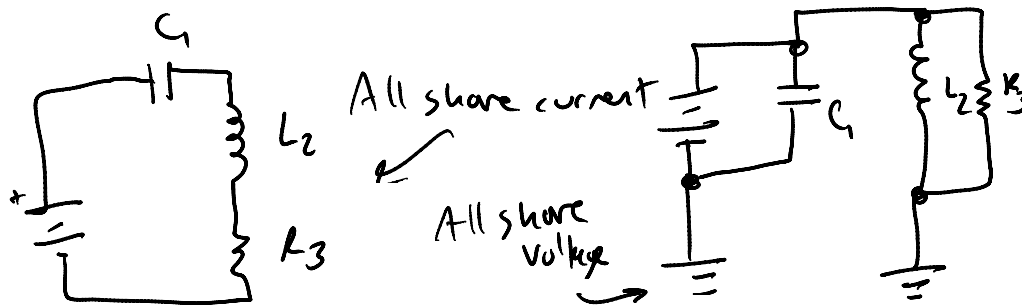
Inductor wants zero V & const. current in steady state

Capacitor wants const. voltage & zero current in steady state.

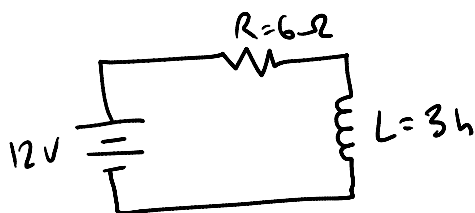
$$V = Li'$$

$$i = \frac{\int V dt}{L}$$

Current in an inductor cannot change instantly.



$$i = \frac{1}{L} \int V dt \text{ is similar to } v = \frac{1}{m} \int F dt$$



KVL:

$$-V_B + V_R + V_L = 0$$

$$Ri + \frac{di}{dt}L = V_B$$

Non-homogeneous

Homogeneous part

$$Ri + \frac{di}{dt}L = 0$$

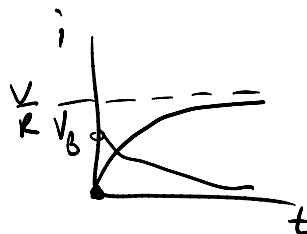
$$e^{-\frac{R}{L}t} = y$$

$$i = a + be^{-\frac{R}{L}t}$$

find a & b

$$i = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$$

In steady state, $\frac{di}{dt} = 0$

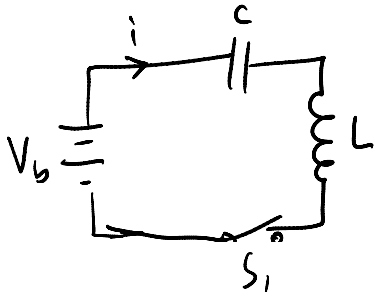


$$V_L = L \frac{di}{dt}$$

$$Ri + L \frac{di}{dt} = V_B = 12 \quad \frac{di}{dt} = \frac{V}{L} = 4$$

$$\frac{di}{dt} \approx 1$$

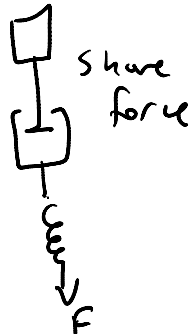
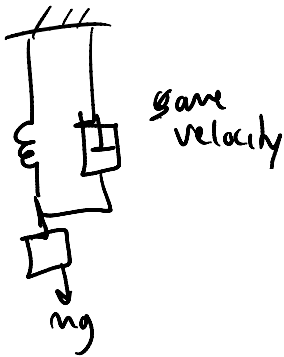
Current in inductor is state variable



KCL: $i_C = i_L = i_B = i$
 KVL: $-V_B + V_C + V_L = 0$
 Const. laws: $V_C = \frac{q_C}{C}$
 $V_L = L \frac{di_L}{dt}$

eqns of motion

$$\begin{aligned} i_L' &= \\ V_C' &= \end{aligned}$$

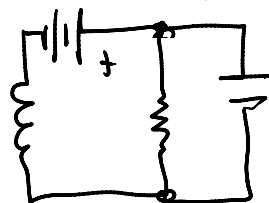


$$\begin{aligned} i' &= \frac{V_B}{L} - \frac{1}{L} V_C \\ V_C &= \frac{1}{C} i \end{aligned}$$

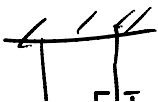
$$\begin{bmatrix} V_C' \\ i' \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{V_B}{L} \end{bmatrix}$$

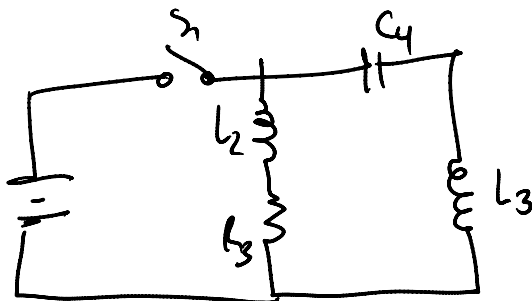
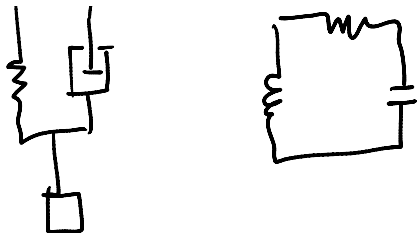


is analogous to



LC system will oscillate
 b/c steady state reached for
 1, & the other doesn't like
 it.



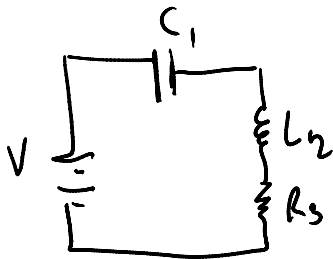


When S_1 first closed,
what is 0?

i_{L2}
 i_{L3}
 V_{R3}
 V_{C4}

Under steady state, C_4 is
full,

Steady state,
 V_{L3}
 V_{L2}
 i_{L3}



$$\text{KVL: } -V_B + V_{C1} + V_{L2} + V_{R3} = 0$$

$$\text{KCL: } i = \text{all } i$$

$$\text{const. laws: } V_{R3} = i R_3$$

$$V_{L2} = L_2 \frac{di}{dt}$$

$$V_{C1} = \frac{q_1}{C_1}$$

$$\text{DEQS: } V_{C1} = \frac{q_1}{C_1} = \frac{i_{C1}}{C_1} = \frac{i_{L2}}{C_1} = \frac{i}{C_1}$$

$$i' = \frac{V_B - V_{C1} - i R_3}{L}$$

$$\text{Natural frequency} = \sqrt{\frac{1}{LC}}$$

Voltage, natural freq

Wednesday, May 16, 2007

7:05 AM



$V = \frac{q}{C}$
 $V_L = L \frac{di}{dt}$
 $L \frac{d^2 q}{dt^2} + \frac{q}{C} = 0$

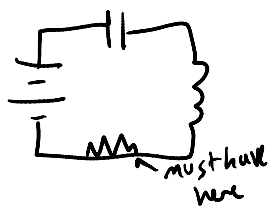
$V_C + V_L = 0$
 $\frac{q}{C} + L \frac{di}{dt} = 0$
 $i = \frac{dq}{dt}$

$E = \frac{1}{2} \frac{q^2}{C}$
 $= \frac{1}{2} C V^2$

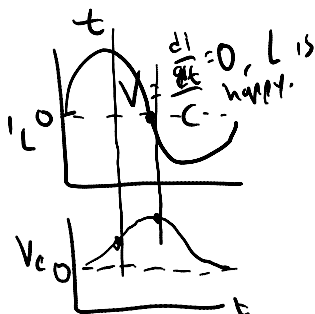
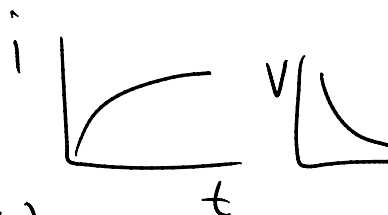
$E = \int_0^q r = \frac{q}{C} dt$

i_L at $t=0$ is 0
 i_L at $t \gg 0$ is (+)
 remove battery,
 i_L is pos. $i_H = -i_L$

$V_C = \frac{d\phi}{dt} \quad \mathcal{P} = \int V_C dt = \int L \frac{di}{dt} dt = \frac{1}{2} L i^2$



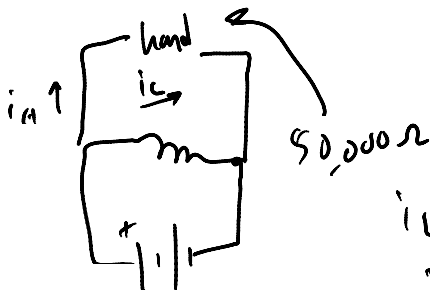
$V_C' = \frac{i}{C}$
 $\frac{di}{dt} = \frac{1}{L} V_C = \frac{1}{L} (V_B - V_C)$



$x' = Ax + B$

$\begin{bmatrix} V_C' \\ i' \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{V_B}{L} \end{bmatrix}$
 Capacitor is happy w/ current = 0

gapped in the middle



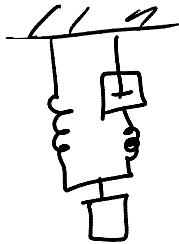
$$i_L' = \frac{V}{L} \quad i_L = t \frac{V}{L}$$

If $i_L = .05 \text{ A}$ before batt removed,
we will get $V_L = 2500 \text{ V}$ after batt
removed

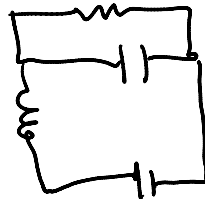
$$\text{Natural frequency} = \sqrt{\frac{1}{LC}}$$

The inductor can accommodate a jump in voltage while the capacitor cannot. The inductor cannot accommodate instantaneous increase in current, and although the capacitor can, since the currents are the same the current at time $=0+$ must still be zero

Pasted from <<http://othello.mech.northwestern.edu/ea3/book/elec9/Lcircuits.htm>>



is analogous to

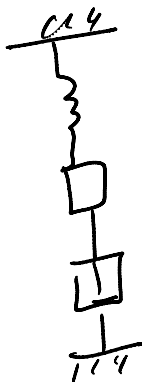


$$V_L + V_{C1} + V_{C2} = 0$$

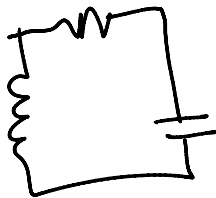
$$V_{C1} = V_{R1}$$

$$V_L = -(V_{C1} + V_{C2})$$

$$L \frac{di}{dt} = -(V_{C1} + V_{C2})$$

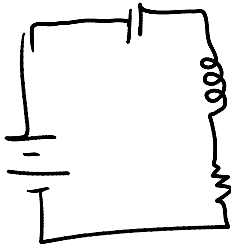


=



Resonance

Monday, May 21, 2007
7:05 AM



$$\text{KVL} = -V_s + V_C + V_L + V_R = 0$$

$$\text{KCL} = i, \text{ only 1 current}$$

Constitutive Laws:

$$V_R = iR$$

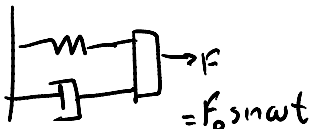
$$V_L = L \frac{di}{dt}$$

$$V_C = \frac{q}{C}$$

DEQs:

$$V_C' = \frac{ic}{C} = \frac{ic}{C} = \frac{i}{C}$$

$$i' = \frac{V_s - V_C - iR}{L}$$

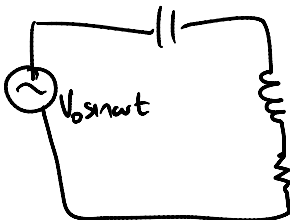


Sinusoidal input

$$X_s' = v_m$$

$$v_m' = \frac{F_0 \sin wt - kX_s - b v_m}{m}$$

$$X_s'' = \frac{F_0}{m} \sin wt - \frac{b}{m} X_s' - \frac{k}{m} X_s$$



$$V_C' = \frac{1}{C} i$$

$$i' = \frac{1}{L} (V_0 \sin wt - V_R - V_C)$$

$$= \frac{1}{L} (V_0 \sin wt - iR - V_C)$$

$$i'' = \left(\frac{\omega V_0}{L} \right) \cos wt - \frac{R}{L} i' - \frac{1}{LC} i$$

$$i'' + \frac{R}{L} i' + \frac{1}{LC} i = \frac{\omega V_0}{L} \cos wt$$

$$i = A \sin wt + B \cos wt$$

$$i' = A \omega \cos wt - B \omega \sin wt$$

$$i'' = -A \omega^2 \sin wt - B \omega^2 \cos wt$$

$$B = \frac{1 - RA}{\frac{1}{C} - L\omega}$$

$$A = \left[R + \frac{1}{R} \left(\frac{1}{\omega C} - L\omega \right)^2 \right]^{-1}$$

$$i = \frac{1}{R + \frac{1}{R} \left(\frac{1}{\omega C} - L\omega \right)^2} \sin wt + \frac{1 - RA}{\omega C - L\omega} \cos wt$$

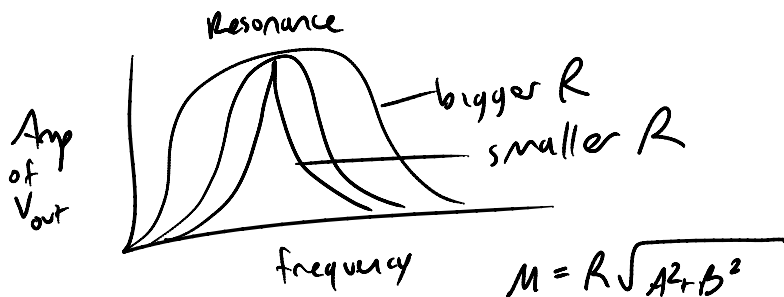
$$1 \dots \dots \frac{1}{\omega C - L\omega} \text{ or } \omega = \sqrt{1/LC}$$

when $L\omega = \frac{1}{\omega C}$ or $\omega = \sqrt{\frac{1}{LC}}$

$i = \max$
 $V_R = iR = \max$

Resonance occurs when the excitation frequency = the natural frequency of the sys.

$\omega = \frac{1}{\sqrt{LC}}$ Due to R dissipation, ω doesn't actually go to infinity. It goes to max.



Something is either resonant or not resonant

$$x_s'' = \frac{k}{m}x_s' + \frac{k}{m}x_s = 0$$

$x = C \exp(rt)$
 $r^2 + \frac{k}{m}r + \frac{k}{m} = 0$

$$i'' + \frac{R}{L}i' + \frac{1}{LC}i = 0$$

$i = C \exp(rt)$
 $r^2 + \frac{R}{L}r + \frac{1}{LC} = 0$

$$r = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(\frac{1}{LC}\right)}}{2}$$

$$r = \frac{-\frac{R}{L} \pm \sqrt{\frac{4}{LC} - \left(\frac{R}{L}\right)^2}}{2}$$

If $\left(\frac{R}{L}\right)^2 < \frac{4}{LC}$, r will have

imaginary components. There will be sinusoidal motion.

$$i = \exp(-r_r t) [A \cos(r_i t) + B \sin(r_i t)]$$

$$= -\frac{R}{2L} \pm \sqrt{\frac{4}{LC} - \left(\frac{R}{L}\right)^2} = r_r \pm r_i$$

Motion decays

Stop sign vibrates in the wind at 10 Hz. What effects play the role of spring & mass?

C? m (spring)

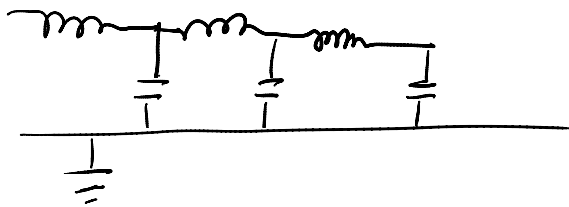
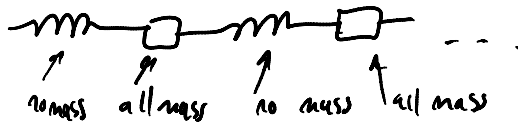
L? $\square$ (mass) mass of stop sign.

Or...

Organ pipes vibrate at 100 - 1000 Hz depending on length.
What effects play role of spring or mass?

Transmission Line

- lumped parameter model for continuous sys.

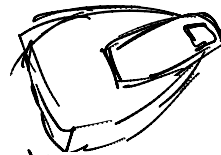


$$V_{m2}' = \frac{k}{m} (-x_{k2} + x_{k3})$$

$$x_{k2}' = V_{m2} - V_{m1}$$

Apply to others, $x_{ki}' = V_{mi} - V_{mi-1}$

$$V_{mi}' = \frac{k}{m} (-x_{ki} + x_{ki+1})$$



Normal Modes

Tuesday, May 22, 2007
7:04 AM

$$x_i' = v_i - v_{i-1}$$

$$v_i' = \left(\frac{k}{m}\right)(-x_i + x_{i+1})$$



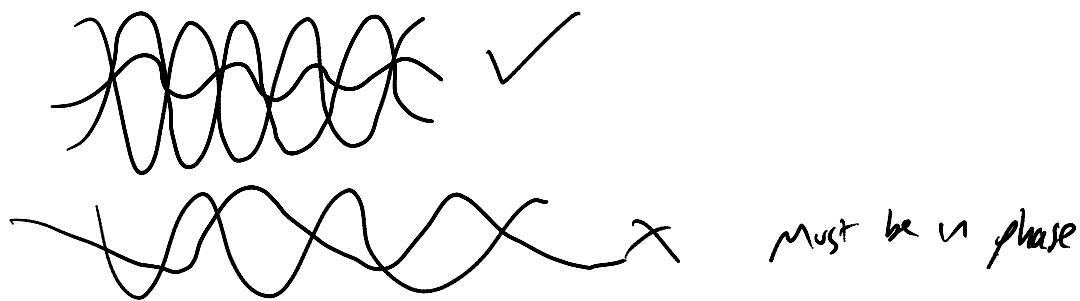
$$x' = \begin{bmatrix} x_1 \\ \vdots \\ x_n \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 & \parallel 0 \\ \parallel 0 & 0 \end{bmatrix} x$$

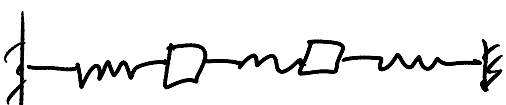
$\begin{matrix} A \\ \curvearrowright \end{matrix}$

Normal Modes

- Masses move in "sync" w/ each other
- Complex motion: superposition of normal modes
- Natural frequencies (from k_i & m_i vals)

which are normal modes?



How many ind. states? 

(4)

$x = \{x_1, x_2, v_1, v_2\}$ which eqn is useful to eliminate x_3 ?

$$x_1 + x_2 + x_3 = 0$$

$$x_1 + x_2 + x_3 = \text{const}$$

$$V_{m1}' = \left(\frac{1}{m_1}\right)(-k_1 x_1 + k_2 x_2) \quad \text{Find } V_{m2}':$$

$$V_{m2}' = \left(\frac{1}{m_2}\right)(-k_2 x_2 + k_3(-x_1 - x_2))$$

x_{k3}' is not needed.

$$x_{k3}' = -V_{m2}$$

$$x_{k1}' = V_{m1}$$

$$x_{k2}' = V_{m2} - V_{m1}$$

$$V_{m1}' = \frac{1}{m_1}(-k_1 x_{k1} + k_2 x_{k2})$$

$$V_{m2}' = \frac{1}{m_2}(-k_2 x_{k2} + k_3(-x_{k1} - x_{k2}))$$

put in matrix form

$$\begin{Bmatrix} x_{k1}' \\ x_{k2}' \\ V_{m1}' \\ V_{m2}' \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ -k_1/m_1 & k_2/m_1 & 0 & 0 \\ -k_3/m_2 & -k_2/m_2 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x_{k1} \\ x_{k2} \\ V_{m1} \\ V_{m2} \end{Bmatrix}$$

$$V_{m1}'' = \frac{1}{m_1}(-k_1 V_{m1} + k_2(V_{m2} - V_{m1}))$$

$$V_{m2}'' = \frac{1}{m_2}(-k_2(V_{m2} - V_{m1}) - k_3 V_{m2})$$

$$\begin{Bmatrix} v_{m1}'' \\ v_{m2}'' \end{Bmatrix} = \begin{bmatrix} -\frac{(k_1+k_2)}{m_1} & \frac{k_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{(k_2+k_3)}{m_2} \end{bmatrix} \begin{Bmatrix} v_{m1} \\ v_{m2} \end{Bmatrix}$$

To solve this prob we need:

4 initial conditions $v_{m1}(0), v_{m2}(0), v_{m1}'(0), v_{m2}'(0)$

Which solns are reasonable for

$$\vec{X} = \begin{Bmatrix} v_{m1} \\ v_{m2} \end{Bmatrix} \quad \text{Eq: } \vec{X}'' = A\vec{X}$$

$$v_{m1} = b_1 \exp(\alpha t) \quad v_{m2} = b_2 \exp(\alpha t)$$

$$x = b \exp(\beta t)$$

$$x = b_1 \exp(\alpha t) + b_2 \exp(\beta t)$$

Describe the soln:

Free vibration

Amps determined by init. conditions & sys parameters

Eigenvalues

Wednesday, May 23, 2007

7:05 AM

$$Ax = \lambda x$$

$$A = \begin{bmatrix} 17 & -6 \\ 45 & -16 \end{bmatrix}$$

$$(A - \lambda I) = 0$$

$$\begin{bmatrix} 17-\lambda & -6 \\ 45 & -16-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - \lambda - 2 = 0$$

$$(\lambda - 2)(\lambda + 1) = 0$$

$$\lambda_1 = 2$$

$$\lambda_2 = -1$$

$$\begin{pmatrix} 17-2 & -6 \\ 45 & -16-2 \end{pmatrix} \begin{pmatrix} (x_1)_1 \\ (x_1)_2 \end{pmatrix} = 0$$

$$\begin{cases} 15(x_1)_1 - 6(x_1)_2 = 0 \\ 45(x_1)_1 - 18(x_1)_2 = 0 \end{cases} \left. \vphantom{\begin{cases} 15(x_1)_1 - 6(x_1)_2 = 0 \\ 45(x_1)_1 - 18(x_1)_2 = 0 \end{cases}} \right\} \text{same not lin. ind.}$$

$$5(x_1)_1 - 2(x_1)_2 = 0$$

$$x_1 = \beta_1 \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} (x_1)_1 \\ (x_1)_2 \end{pmatrix}$$

$$\uparrow \lambda_1 = 2$$

$$\begin{pmatrix} 17+1 & -6 \\ 45 & -16+1 \end{pmatrix} \begin{pmatrix} (x_2)_1 \\ (x_2)_2 \end{pmatrix} = 0$$

$$\begin{cases} 3(x_2)_1 - (x_2)_2 = 0 \\ x_2 = \beta_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} \leftarrow \lambda_2 = -1 \end{cases}$$

If $A \rightarrow \lambda_i$
 $A' = A^T \rightarrow \lambda_i$ same Not same eigenvectors

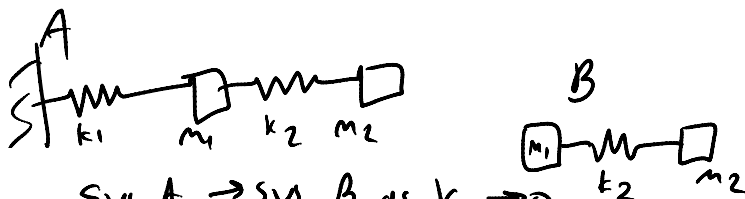
If $A \rightarrow \lambda_i$
 $A^{-1} \rightarrow \frac{1}{\lambda_i}$ But same eigenvectors

If $A \rightarrow \lambda_i$ But same eigenvectors
 $\alpha A \rightarrow \alpha \lambda_i$

① $|A| = 2(-1) = -2$

② Trace $A = \sum \text{eigenvalues}$
 $= \lambda_1 + \lambda_2 + \dots$

$\begin{pmatrix} 1 & -6 \\ 45 & -16 \end{pmatrix}$ Trace $A = +1$
 $\lambda_2 + \lambda_1 = 2 - 1 = +1$ ✓



sys A $\rightarrow$ sys B as $k_1 \rightarrow 0$

sys A has 2 normal modes How many modes does sys B have? ② the sys will move as a whole or move stationary.

Team A: Find state eqns for sys A
 Use state vector $\vec{X} = \{V_{m1}, V_{m2}, X_{k1}, X_{k2}\}$

$$V_{m1}' = \frac{1}{m_1} (-k_1 X_{k1} + k_2 X_{k2})$$

$$V_{m2}' = \frac{1}{m_2} (-k_2 X_{k2})$$

$$X_{k1}' = V_{m1}$$

$$X_{k2}' = V_{m2} - V_{m1}$$

→ $V_{m1}' = \frac{1}{m_1} (0 + k_2 X_{k2})$

$$V_{m2}' = \frac{1}{m_2}(-k_2 x_{k2})$$

$$x_{k2}' = V_{m2} - V_{m1}$$

$$V_{m1}'' = \frac{1}{m_1}(-\cancel{k_1} + k_2)V_{m1} + k_2 V_{m2}$$

$$V_{m2}'' = \left(\frac{k_2}{m_2} - \frac{k_2}{m_2} \right) V_{m2}$$

$$\begin{Bmatrix} V_{m1}'' \\ V_{m2}'' \end{Bmatrix} = \begin{bmatrix} -\frac{k_1+k_2}{m_2} & \frac{k_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} \end{bmatrix} \begin{Bmatrix} V_{m1} \\ V_{m2} \end{Bmatrix}$$

$$\vec{x}'' = [A] \vec{x}$$

Try a soln in the form of

$$\vec{x} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} \exp \alpha t$$

$$\vec{x} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} \quad \vec{x} = \vec{v} \exp(\alpha t) \quad \vec{v} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

which of the following are = to $x'' = [A] \vec{x}$:

$$\alpha^2 \vec{v} = [A] \vec{v}$$

$$([A] - \alpha^2 [I]) \vec{v} = \vec{0}$$

$$\begin{bmatrix} A_{11} - \alpha^2 & A_{12} \\ A_{21} & A_{22} - \alpha^2 \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\rightarrow \alpha^2 = -\omega^2$$

And $\vec{v}$

$$x'' = [A] \vec{x}$$

$$x = \vec{v} \exp(\alpha t)$$

$$\text{Get } \alpha^2 \vec{v} = [A] \vec{v} \quad \text{or } ([A] - \alpha^2 [I]) \vec{v} = \vec{0}$$

$$c_1 \sin(\omega t) + c_2 \cos(\omega t) = 0$$

Satisfied iff $\det([A] - \alpha^2[I]) = 0$

Find α^2 vals (2 for 2×2 , 3 for $3 \times 3 \dots$)

Put back into eqn to get constraint on $\vec{v}$
Result: pairs of α, v , one for each mode.

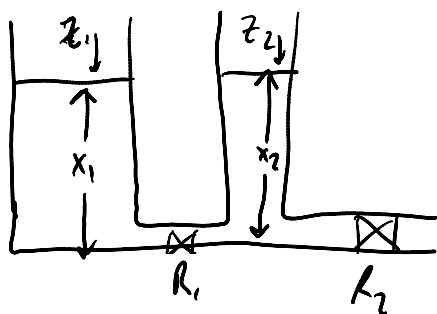
α^2 vals are neg: α 's are complex

For each $\vec{v}$, we know it's dirn, not magnitude.

Sols are thus

$$\vec{x} = c_1 \vec{v}_1 e^{\alpha_1 t} + c_2 \vec{v}_2 e^{\alpha_2 t}$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



$$q_1(t) = \frac{x_1 - x_2}{R_1}$$

$$q_2(t) = \frac{x_2}{R_2}$$

$$\frac{dM}{dt} = \sum \dot{m}_i - \sum \dot{m}_e$$

$$\dot{x}_1 = \left[-\frac{1}{A_1 R_1} \right] x_1 + \frac{1}{A_1 R_1} x_2 + \frac{z_1}{A_1}$$

$$\dot{x}_2 = \left[\frac{1}{A_2 R_1} \right] x_1 - \left(\frac{1}{A_2 R_1} + \frac{1}{A_2 R_2} \right) x_2 + \frac{z_2}{A_2}$$

$$\dot{\bar{x}} = A \bar{x} + B \bar{z}$$

$$A = \begin{bmatrix} \frac{1}{A_1 R_1} & \frac{1}{A_1 R_2} \\ \frac{1}{A_2 R_1} & \frac{1}{A_2 R_2} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{A_1} & 0 \\ 0 & \frac{1}{A_2} \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A_1 = A_2 = 10 \text{ m}^2$$

$$R_1 = R_2 = .1 \text{ s/m}^2$$

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} .1 & 0 \\ 0 & .1 \end{bmatrix}$$

$$\begin{bmatrix} -1-\lambda & 1 \\ 1 & -2-\lambda \end{bmatrix} = 0$$

$$\lambda^2 + 3\lambda + 1 = 0$$

$$\lambda_1 = .382$$

$$\lambda_2 = -2.618$$

Sohrab can't write large enough

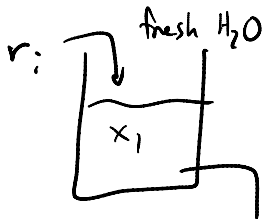
$$c_1 \begin{bmatrix} \end{bmatrix} e^{-.382t} + c_2 \begin{bmatrix} \end{bmatrix} e^{-2.618t}$$

$$(A - \lambda I)x = 0$$

$$\vec{v}_1 = \beta \begin{bmatrix} 1 \\ .619 \end{bmatrix}$$

$$\vec{v}_2 = \beta_2 \begin{bmatrix} 1 \\ -1.618 \end{bmatrix}$$

$$x = c_1 \begin{bmatrix} 1 \\ .618 \end{bmatrix} e^{-.392t} + c_2 \begin{bmatrix} 1 \\ -1.618 \end{bmatrix} e^{-2.518t}$$



x_i = # kg salt in tank i

r_i = m³/sec volumetric flowrate

$$\begin{aligned} \frac{dM}{dt} &= \sum \dot{m}_i - \sum \dot{m}_o \\ &= 0 - \sum \dot{m}_o = -\dot{m}_o \\ &= \rho \dot{V}_i = -\dot{V} \frac{M_i}{V} \\ &= -\frac{\dot{V}}{V} x_i = -k x_i \end{aligned}$$

$$\vec{x}' = \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix}$$

$$r_i = \dot{V}$$

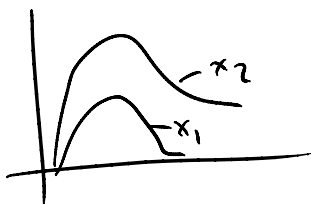
$$x' = Ax$$

$$x_1' = -k_1 x_1$$

$$x_2' = k_1 x_1 - k_2 x_2$$

$$x_3' = k_2 x_2 - k_3 x_3$$

$$A = \begin{bmatrix} -.5 & 0 & 0 \\ .5 & -.25 & 0 \\ 0 & .25 & -.2 \end{bmatrix}$$



$$\text{initial condition } x_0 = [1.5; 0; 0]$$

$$|A - \lambda I| = 0$$

$$\begin{matrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{matrix}$$

15 kg salt in tank

$$[V, E] = \text{eig}(A)$$

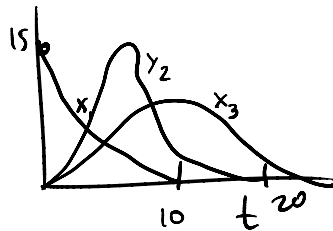
$$V = \begin{bmatrix} 0 & 0 & 3586 \\ 0 & .194 & -.7171 \\ 1 & -.9806 & .5976 \end{bmatrix}$$

$$E = \begin{bmatrix} -.2 & 0 & 0 \\ 0 & -.25 & 0 \\ 0 & 0 & -.5 \end{bmatrix} \quad \begin{aligned} \lambda_1 &= -.2 \\ \lambda_2 &= -.25 \\ \lambda_3 &= -.5 \end{aligned}$$

$$X = c_1 \vec{v}_1 e^{-.2t} + c_2 \vec{v}_2 e^{-.25t} + c_3 \vec{v}_3 e^{-.5t}$$

$$x_i = \sum c_j \vec{v}_j e^{-\lambda_j t}$$

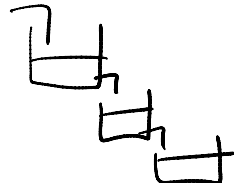
Plot $[t, x_1, t, x_2, t, x_3]$



Eigenvalues

Tuesday, May 29, 2007
7:08 AM

$$\begin{aligned}x_1' &= -k_1 x_1 \\x_2' &= k_1 x_1 - k_2 x_2 \\x_3' &= k_2 x_2 - k_3 x_3\end{aligned}$$



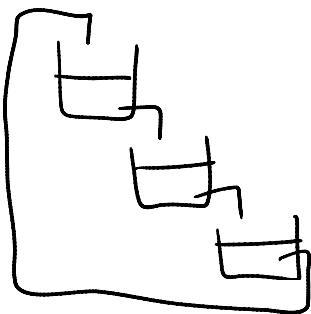
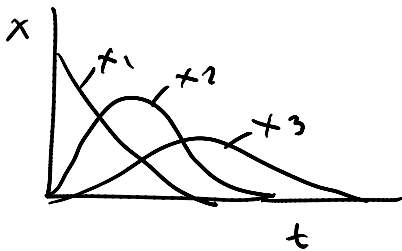
$$A = \begin{bmatrix} -0.5 & 0 & 0 \\ 0.5 & -0.25 & 0 \\ 0 & 0.25 & -0.2 \end{bmatrix} \quad \vec{x}' = A \vec{x} \quad x_0 = [15; 0; 0]$$

$$[V, E] = \text{eig}(A)$$

$$V = \begin{bmatrix} 0 & 0 & 0.3586 \\ 0 & 0.1861 & 0.0717 \\ 1 & 0.98 & 0.59 \end{bmatrix}$$

$$E = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.25 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}$$

$$\vec{x} = c_1 \vec{v}_1 e^{-0.2t} + c_2 \vec{v}_2 e^{-0.25t} + c_3 \vec{v}_3 e^{-0.5t}$$



Steady state val

$$\begin{aligned}x_1' &= k_3 x_3 - k_1 x_1 \\x_2' &= k_1 x_1 - k_2 x_2 \\x_3' &= k_2 x_2 - k_3 x_3\end{aligned}$$

$$A = \begin{bmatrix} -k_1 & 0 & k_3 \\ k_1 & -k_2 & 0 \\ 0 & k_2 & -k_3 \end{bmatrix} \quad E = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.25 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}$$

$$A = \begin{bmatrix} k_1 & -k_2 & 0 \\ 0 & k_2 & -k_3 \end{bmatrix} = \begin{bmatrix} .2 & -.4 & 0 \\ 0 & .4 & -.2 \end{bmatrix}$$

$$V = \begin{bmatrix} -.6667 & .447 - .048i \\ -.383 & .048 - .417i \\ -.667 & -.56 + .048i \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -.4 + .2i & 0 \\ 0 & 0 & -.4 - .2i \end{bmatrix}$$

$$\vec{X}_i = \sum c_i V_i e^{-\lambda_i t}$$

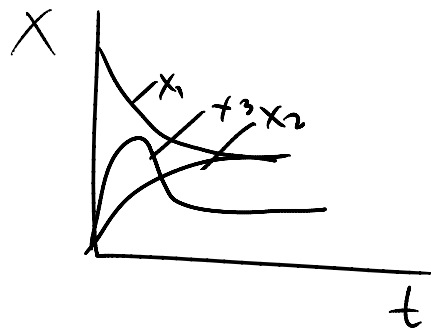
$$V_i = X_0$$

$$C = \frac{V}{\lambda}, c'$$

$$c_1 = -6.7i$$

$$c_2 = 5.77 + 2.57i$$

$$c_3 = 5.77 - 2.57i$$



For oscillation of a simple pendulum,

There is 1 natural frequency

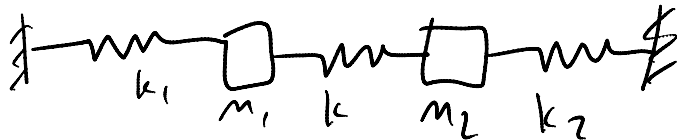
It is similar to a spring-mass sys

Depends on the length of the pendulum but not on the mass.

Restoring force of gravity is like a spring

Springs

Wednesday, May 30, 2007
7:04 AM



$$x_{k1} + x_k + x_{k2} = 0$$

$$x_{k1}' = v_{m1}$$

$$x_{k2}' = -v_{m2}$$

$$v_{m1}' = \frac{1}{m_1} (-k_1 x_{k1} + k(-x_{k1} - x_{k2}))$$

etc

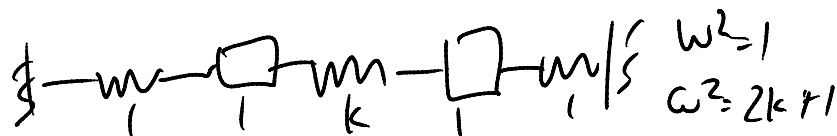
$$\vec{x} = \begin{bmatrix} v_{m1} \\ v_{m2} \end{bmatrix}$$

$$\vec{x}'' = \begin{bmatrix} -1-k & k \\ k & -1-k \end{bmatrix} \vec{x}$$

$$\det \nearrow = 0$$

$$x \Big|_{\cos t} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} c_1 \cos t + \begin{bmatrix} 1 \\ -1 \end{bmatrix} c_2 \cos(\sqrt{2k+1}t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} d_1 \sin t + \begin{bmatrix} 1 \\ -1 \end{bmatrix} d_2 \sin(\sqrt{2k+1}t)$$

which activate normal modes?



- A) $c_1 = d_1 = 1, c_2 = d_2 = 0$
- B) $c_1 = 1, d_1 = 1, c_2 = d_2 = 0$
- C) $c_1 = d_1 = 0, c_2 = 6, d_2 = 0$

Any IC that
leads to only
a single
frequency is
active

achue

Sloshing occurs when eigenvals are close.

$$\text{Spring-mass-spring-mass} \quad x = \begin{bmatrix} v_{m1} \\ v_{m2} \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix} (a_1 \cos \omega_1 t + b_1 \sin \omega_1 t) + \begin{bmatrix} 4 \\ -2 \end{bmatrix} (a_2 \cos \omega_2 t + b_2 \sin \omega_2 t)$$

Which initial conditions would work to excite only normal mode 1?

Which would resonate w/ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$?

$$x(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} -1 \\ -2 \end{bmatrix} \quad \text{anything relating to}$$

$$x(0) = \begin{bmatrix} 4 \\ 8 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Let $v'_{m1}(0) = v'_{m2}(0) = 0$
for all

$$\text{Spring-mass-spring-mass-spring-mass-spring-mass}$$

How many ind. state vars for n masses & $n+1$ springs?

$$(2n)$$

$$v_{m2} + x_{k2} = \dots ?$$

$$V_{m_2}' = \frac{k}{n} (-x_{k_2} + x_{k_3})$$

$$x_{k_2}' = V_{m_2} - V_{m_1}$$

$$n=4$$

exceptions

$$x_{k_1}' = V_{m_1}$$

$$x_{k_{(n+1)}}' = -V_{m_n}$$

$$V_{m_i}'' = \frac{k}{n} (V_{m_{i+1}} + V_{m_{(i-1)}} - 2V_{m_i})$$

Works for $i = 2, \dots, n-1$

$$V_{m_i}'' = \frac{k}{n} (V_{m_{i+1}} + V_{m_{i-1}} - 2V_{m_i}) \text{ for } i = 2, \dots, n-1$$

$$\text{exceptions} - V_{m_1}'' = \frac{k}{n} (V_{m_2} - 2V_{m_1})$$

$$V_{m_{(n)}}'' = \frac{k}{n} (V_{m_{(n-1)}} - 2V_{m_{(n)}})$$

$$x'' = Ax, \text{ pos + neg elmts of } A \text{ are}$$

