Fundamental Concepts

Wednesday, January 03, 2007
8:01 AM

From text
HW #1: 1.6, 1.20, 1.24, 1.30 Due Mon
Pablo Durango-Cohen
Use graph paper for HW
Office Hours T 1600–1700, W 1600–1700
TA office hrs in MG 47

Basic tools, Fundamental quantities / units VECTORS
Fundamental physics concepts
Statics
Dynamics

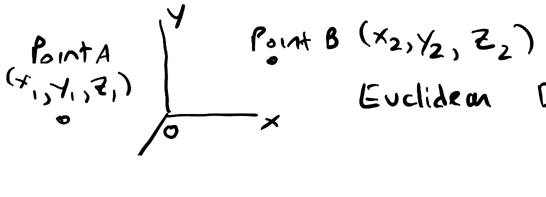
Mechanics – study of the effects of forces and moments
The 1st branch of engineering to become formal
Newton's Laws began transformation of basic sciences into engineering

Measurments & sig figs
Numbers that we write should reflect the certainty that we have in them

We have a finite space to save info on a comp. It's very precise now because we have a lot more space. However, we can't rely on excessive accuracies b/c you need not take up more space than necessary.

Sig figs refer to the #s after leading zeros. 0.343, 343E2, 34300 is diff from 343E2 b/c we are expres them up to certain digits.

Fundamental quantities –
- Distance
  * Space – A set where we can define a <u>distance</u> measure between the elements

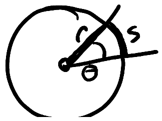


Euclidean Distance: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

z'

SI units are Meters
US customary units are feet

Angles



$s = \theta r$, θ is in radians
degrees sometimes

Newton's Laws, Quantities

Thursday, January 04, 2007

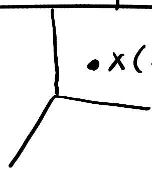
8:00 AM

Fundamental Quantities

	SI	US
Distance	m	ft
Angles	rad	deg

Time - interval between consecutive units

Time	s	s



$\bullet\, x(t)$ $\quad \frac{d\,x(t)}{dt}$ or $x'(t)$ in units $\frac{\text{distance}}{\text{time}}$

Velocity

Velocity	m/s	ft/s

$x''(t)$ or $\left(\frac{\frac{d\,x(t)}{dt}}{dt}\right)$ Acceleration

Acceleration	m/s^2	ft/s^2

Mass - The amount of matter that composes an object

Mass	kg	slug

Newton's Laws

1. When the sum of forces acting on a particle is 0, its velocity is constant.
   → If the particle is initially stationary, it will remain stationary
2. When the sum of forces acting on a particle is not 0, the sum of forces is equal to the rate of change of the linear momentum of a particle.

$p(t) \equiv m(t)\,v(t)$ Momentum kg - m/s

Momentum	kg·m/s	slug·ft/s

$$F = \frac{\partial}{\partial t} p(t) = m'(t)v(t) + m(t)\,v'(t)$$

$m(t) = m$, constant $\quad m'(t) = 0$

$F = m(t)\,v'(t) \qquad v'(t) = a(t)$

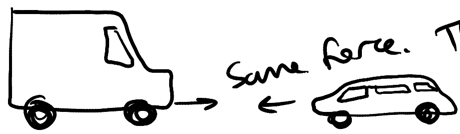
$F = m(t)\,a(t)$

$kg \cdot m/s^2 = N$, Newtons

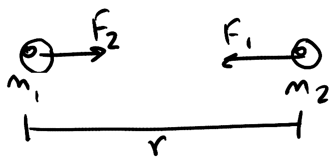
Force	N	lb

1 Newton = external force needed to obtain $1 m/s^2$ if applied to 1 kg

3. The forces exerted by two particles on each other are equal in magnitude and opposite in direction.



Same force. They withstand diff forces b/c they are diff sizes.



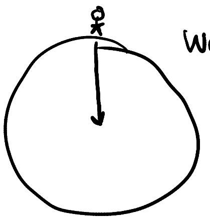
Two spheres/particles

$$F_1 = -F_2 = F$$

$$F = G\frac{m_1 m_2}{r^2} \qquad G = 6.67E{-11}$$

to find units of G, you do this:

$$G = \frac{F \cdot r^2}{m_1 m_2} \qquad \frac{N \cdot m^2}{kg^2}$$



Weight of the person = $G\frac{m_1 m_2}{r^2}$

$= m_p \cdot g \qquad g = 9.81\ m/s^2$ on Earth, at sea level

$= 32.2\ ft/s^2$

The radius of earth changes depending on where you are located. The larger it is, the smaller g is. The smaller it is (below sea level) the larger g is.

$$F = m_p a \qquad W = m_p g$$

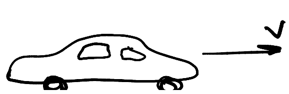
$$\frac{F}{W} = \frac{m_p a}{m_p g} = \frac{a}{g}$$

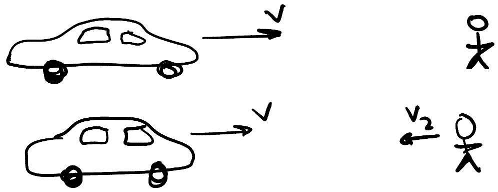
$$a = \frac{F}{W} g \qquad 3\ g\text{'s},\ 6\ g\text{'s},\ 8\ g\text{'s} \ldots$$

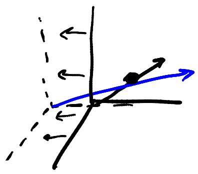
Scalars & Vectors

<u>Scalar</u> - a quantity that is completely described by a real #.

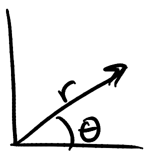
<u>Vector</u> - a quantity that needs to be described with respect to a frame of reference (magnitude, angle)





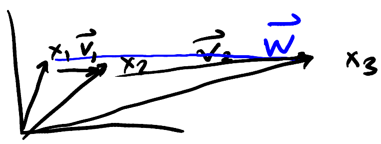


The vector changes depending on where the origin is.



Vector Addition

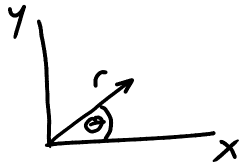
$\vec{w} = \vec{v}_1 + \vec{v}_2$



Triangle rule

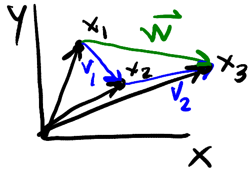
Vectors, Cmpnts

Friday, January 05, 2007
8:00 AM



$|\vec{r}|$ or r <u>magnitude</u>
θ is direction.

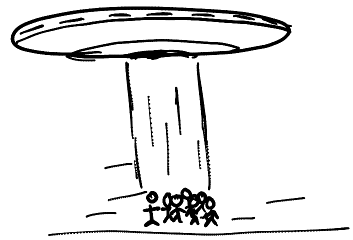
<u>Vector addition</u>



Triangle rule for vector addition.

Commutative Property

$$\vec{w} = \vec{v}_1 + \vec{v}_2 = \vec{v}_2 + \vec{v}_1$$



Associative Property

$$\vec{w} = (\vec{v}_1 + \vec{v}_2) + \vec{v}_3 = \vec{v}_1 + (\vec{v}_2 + \vec{v}_3)$$

Multiplication of a vector and a scalar

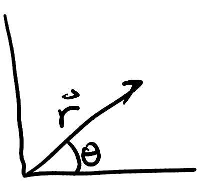
$$\vec{s} = a \cdot \vec{r}$$

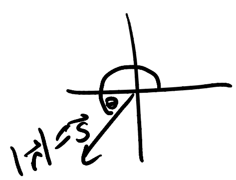
$$\Rightarrow |\vec{s}| = a \cdot |\vec{r}|$$

$$\Rightarrow \theta_s = \theta_r$$

$a = -1$, $|\vec{s}| = -|\vec{r}|$; $\theta_s = \theta_r$

$|\vec{s}| = |\vec{r}|$, but the direction is opposite. $\theta_s = \theta_r + \pi$



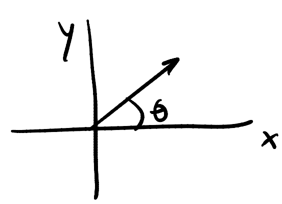


Additive inverse

$\vec{r}$ $\exists$ $\hat{r}$ such that $\vec{r} + \hat{r} = \vec{0}$

↑ there exists

$\hat{r} = -\vec{r}$



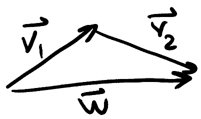
<u>Unit vectors</u> - a vector of magnitude 1

$\vec{U}$ vector, $\vec{e}_U$ unit vector in U direction. $\vec{e}_U = \frac{1}{|\vec{U}|}\vec{U}$

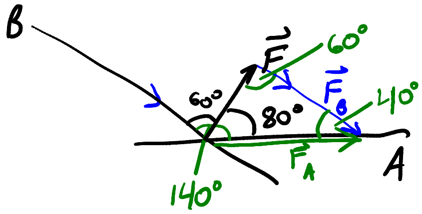
Obtaining unit vectors in a given direction allows us to express any vector in that direction as a multiple of the unit vector

Vector components

A set of vectors that add up to a vector of interest



$\vec{w}$ is a net vector. $\vec{v}_1 + \vec{v}_2 = \vec{w}$



$|\vec{F}| = 400$ lbs

$\vec{F}_B \parallel B$

$\vec{F}_A \parallel A$

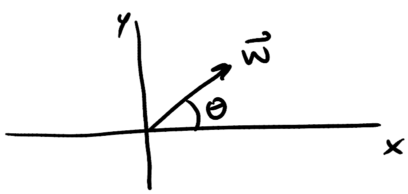
Law of Sines

$$\frac{|\vec{F}|}{\sin 40} = \frac{|\vec{F}_A|}{\sin 60}$$

$|\vec{F}_A| = 539$ lbs

by same process $|\vec{F}_B| = 613$ lbs

Cartesian Components



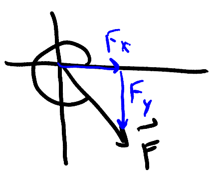
$\vec{w} = \vec{w}_x + \vec{w}_y$

$\vec{w} = |\vec{w}_x|\hat{i} + |\vec{w}_y|\hat{j}$

$|\vec{w}_x| = |\vec{w}|\cos\theta$

$|\vec{w}_y| = |\vec{w}|\sin\theta = |\vec{w}|\cos(\frac{\pi}{2}-\theta)$

$|\vec{w}|^2 = |\vec{w}_x|^2 + |\vec{w}_y|^2$

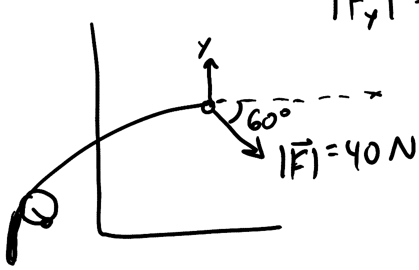


$|\vec{F}| = 165$ MN Mega Newtons

$|\vec{F}_x| = 130$ MN

$|\vec{F}_y|^2 = |\vec{F}|^2 - |\vec{F}_x|^2$

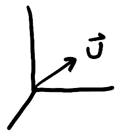
$|\vec{F}_y| = 102$ MN



$|\vec{F}_x| = |\vec{F}|\cos 60 = 20$ N

$|\vec{F}_y| = -|\vec{F}|\sin 60 = -35$ N

3 D Vector components



$$\vec{u} = \vec{u}_x + \vec{u}_y + \vec{u}_z$$
$$= |\vec{u}_x|\hat{\imath} + |\vec{u}_y|\hat{\jmath} + |\vec{u}_z|\hat{k}$$

$\Theta_x, \Theta_y, \Theta_z$ direction angles with respect to each axis.

$\cos\theta_x, \cos\theta_y, \cos\theta_z$ direction cosines.

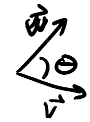
$$|\vec{u}_x| = |\vec{u}|\cos\theta_x$$
$$|\vec{u}_y| = |\vec{u}|\cos\theta_y$$
$$|\vec{u}_z| = |\vec{u}|\cos\theta_z$$
$$|\vec{u}|^2 = |\vec{u}_x|^2 + |\vec{u}_y|^2 + |\vec{u}_z|^2$$
$$|\vec{u}|^2 = |\vec{u}|^2\cos^2\Theta_x + |\vec{u}|^2\cos^2\theta_y + |\vec{u}|^2\cos^2\Theta_z$$
$$|\vec{u}|^2 = |\vec{u}|^2(\cos^2\theta_x + \cos^2\Theta_y + \cos^2\theta_z)$$
$$\boxed{1 = \cos^2\theta_x + \cos^2\Theta_y + \cos^2\theta_z}$$

$$\vec{u} + \vec{w} = (u_x + w_x)\hat{\imath} + (u_y + w_y)\hat{\jmath} + (u_z + w_z)\hat{k}$$

$$\vec{a} + \vec{u} = au_x\hat{\imath} + au_y\hat{\jmath} + au_z\hat{k}$$

Dot Product - an operation between 2 vectors that yields a scalar

$\vec{v} \cdot \vec{w} = |\vec{v}|\,|\vec{w}|\cos\theta$



Ex $\hat{\imath} \cdot \hat{\jmath} = 0$ because $\perp$

$\hat{\imath} \cdot \hat{\imath} = 1$ because $\parallel$ $\quad |\hat{\imath}, \hat{\jmath}, \hat{k}| = 1$

Properties

Commutative $\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$ $\quad \cos(-\theta) = \cos\theta$

Scalar Multiplication $\quad a(\vec{v} \cdot \vec{w}) = a\vec{v} \cdot \vec{w} = \vec{v} \cdot a\vec{w}$

Distributive $\quad \vec{v} \cdot (\vec{w} + \vec{x}) = \vec{v} \cdot \vec{w} + \vec{v} \cdot \vec{x}$

$\vec{v} = V_x \hat{\imath} + V_y \hat{\jmath}$

$\vec{w} = w_x \hat{\imath} + w_y \hat{\jmath}$

$$\vec{v} \cdot \vec{w} = (V_x \hat{\imath} + V_y \hat{\jmath}) \cdot (w_x \hat{\imath} + w_y \hat{\jmath})$$
$$= V_x \hat{\imath} \cdot w_x \hat{\imath} + V_x \hat{\imath} \cdot w_y \hat{\jmath} + V_y \hat{\jmath} \cdot w_x \hat{\imath} + V_y \hat{\jmath} \cdot w_y \hat{\jmath}$$
$$= V_x W_x + 0 + 0 + V_y W_y = V_x W_x + V_y W_y$$

$\vec{v} = 2\hat{\imath} - 3\hat{\jmath} + 8\hat{k}$

$\vec{w} = 5\hat{\imath} - \hat{\jmath} - \hat{k}$

$\vec{v} \cdot \vec{w} = 10(1) + 3(1) - 8(1) = 5$

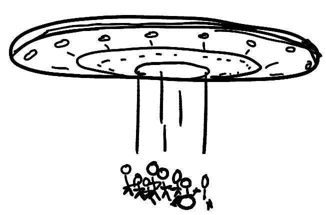
$\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}|\cos\theta$

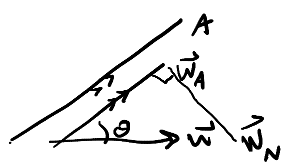
$5 = \left(\sqrt{4+9+64}\right)\left(\sqrt{25+1+1}\right)\cos\theta = \left(\sqrt{77}\right)\left(\sqrt{27}\right)\cos\theta$

$\dfrac{5}{(\sqrt{77})(\sqrt{27})} = \cos\theta$

$84^\circ = \theta$

$\vec{v} \cdot \vec{v} = |\vec{v}||\vec{v}|\cos(0) = |\vec{v}|^2$





$\vec{w} = \vec{w}_A + \vec{w}_N$

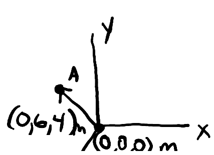
$\hat{e}_A$ unit vector in A direction

$\vec{w} \cdot \hat{e}_A = w_A$ in the $\hat{e}_A$ direction

$\hat{e}_A(\hat{w} \cdot \hat{e}_A) = \vec{w}_A$

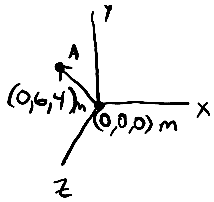
$\vec{w} - \vec{w}_A = \vec{w}_N$

$\vec{F} = 10\hat{\imath} + 12\hat{\jmath} - 6\hat{k}$ (N)



$\vec{A} = 6\hat{\jmath} + 4\hat{k}$

$\hat{e}_A = \dfrac{1}{|\vec{A}|} \cdot A = \dfrac{1}{\sqrt{52}}(6\hat{\jmath} + 4\hat{k})$



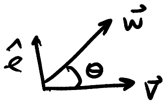
$\vec{A} = 6\hat{j} + 4\hat{k}$

$\hat{e}_A = \frac{1}{|\vec{A}|} \cdot A = \frac{1}{\sqrt{52}}(6\hat{j} + 4\hat{k})$

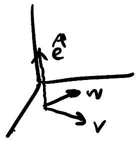
$\vec{F}_A = (\vec{F} \cdot \hat{e}_A)\,\hat{e}_A$

Cross Product

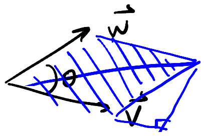
$\vec{v} \times \vec{w} = |\vec{v}||\vec{w}| \sin\theta \; \hat{e}$



$\hat{e}$ = unit vector that is $\perp$ to both $\vec{v}$ & $\vec{w}$. 3D space.



Right Hand Rule



Cross product gives us the area of the ▱.

Properties

Anti-Commutative $\vec{v} \times \vec{w} \neq \vec{w} \times \vec{v}$ $\quad \vec{v} \times \vec{w} = -(\vec{w} \times \vec{v})$

Associative $a(\vec{v} \times \vec{w}) = a\vec{v} \times \vec{w} = \vec{v} \times a\vec{w}$

Distributive $\vec{v} \times (\vec{w} \times \vec{x}) \neq (\vec{v} \times \vec{w}) \times \vec{x}$

$\hat{i} \times \hat{i} = 0$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{i} \times \hat{k} = -\hat{j}$

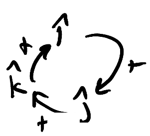
$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

$\hat{k} \times \hat{i} = \hat{j}$



$\vec{v} = 3\hat{i} + 2\hat{j}$

$\vec{w} = 2\hat{i} + 4\hat{j}$

$\vec{v} \times \vec{w} = 0 + 12\hat{k} - 4\hat{k} + 0$

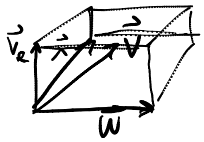
$\vec{v} \times \vec{w} = 8\hat{k}$

Determinant representation

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ V_x & V_y & V_z \\ W_x & W_y & W_z \end{vmatrix} = (V_y W_z - V_z W_y)\,\hat{i} - (V_x W_z - V_z W_x)\,\hat{j} + (V_x W_y - V_y W_x)\,\hat{k}$$

Mixed Triple Product

$$\vec{v} \cdot (\vec{w} \times \vec{x}) = |\vec{v}||\vec{w}||\vec{x}| \sin \theta_w \cos \theta_{ve}$$



Volume of the box

Properties

$\vec{v}\cdot(\vec{w}\times\vec{x}) = \vec{w}\cdot(\vec{x}\times\vec{v}) = \vec{x}\cdot(\vec{v}\times\vec{w})$

Order of factors is not important when computing volume

$\vec{v}\cdot(\vec{w}\times\vec{x}) = -v\cdot(\vec{x}\times\vec{w})$

$\vec{v} = 6\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{w} = 2\hat{i} + 7\hat{j}$

$\vec{x} = 3\hat{i} + 2\hat{k}$

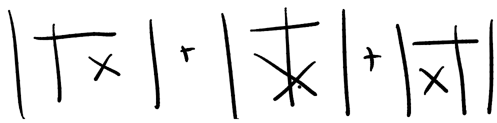
$\vec{v}\cdot(\vec{w}\times\vec{x})$

$\vec{v}\cdot\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 7 & 0\\ 3 & 0 & 2\end{vmatrix} = \vec{v}\cdot\overrightarrow{\text{some vector}}$ and then do dot product.

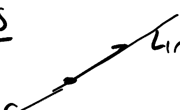
$(6\hat{i} + 2\hat{j} - 4\hat{k})\cdot(14\hat{i} - 4\hat{j} - 21\hat{k}) = 84 \quad -8 \quad +84 = 160$

$\begin{vmatrix}6 & 2 & -4\\ 2 & 7 & 0\\ 3 & 0 & 2\end{vmatrix} = 6(14) - 2(4) + (-4)(-21) = 160$

You can see how you do that.



Terminology + conventions



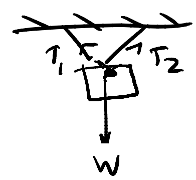
Line of action of a force.

Force - vector

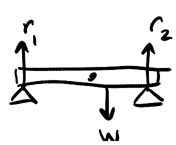
System of forces - sets of various forces

→ Forces in a 2D plane - Coplanar forces

Concurrent forces of action intersect.



Parallel Forces

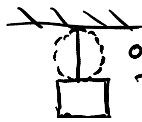


Systems of parallel forces.

External Forces - Forces that are induced or produced by an object that is external.

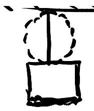
The weight of an object is an external force (depends on planet)

Internal Forces - A force that is induced by an object onto itself.



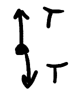
object is rope.

T in rope? Focus on point in rope.



object is rope.

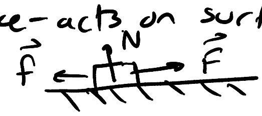
T in rope? Focus on point in rope.



Body force - acts on the volume of an object. weight

$w = mg$
$= \rho v g$

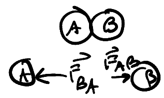
Surface Force - acts on surface of an object. Friction



Fisikes
is $f \mu N$

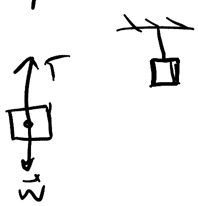
$|\vec{F}| = \mu |\vec{f}|$ $f = \mu N$!

Contact Force - acts on an object because of contact Collisions

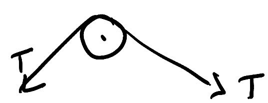


$\vec{F}_{BA} = -\vec{F}_{AB}$

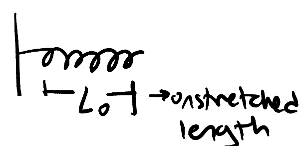
Ropes

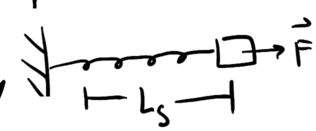


Pulleys



Springs





$|\vec{F}| = k(L_s - L_0)$
k in N/m

<u>Equilibrium</u> - Sum of forces acting on object is 0, velocity is constant. If velocity is 0, it will remain 0.

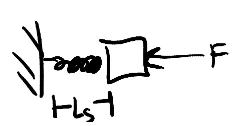
$\Sigma \vec{F} = \vec{0}$, $\vec{v}$ is constant.

When the object is not rotating, it is in equilib.

$(\Sigma \vec{F}_x)\hat{\imath} + (\Sigma \vec{F}_y)\hat{\jmath} + (\Sigma \vec{F}_z)\hat{k} = \vec{0}$

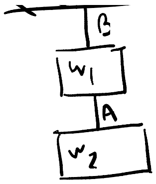
Scalar equilibrium equations.

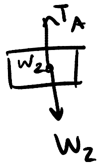
$\Sigma F_x = 0$ $\Sigma F_y = 0$



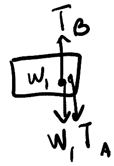
Free body diagram - representation of the <u>external</u> forces that are acting on an object

1) Isolate object of interest
2) sketch object & write info about geometry
3) sketch external forces





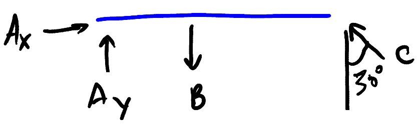
$$\Sigma F_y = 0$$
$$T_A - W_2 = 0$$
$$T_A = W_2$$



$$\Sigma F_y = 0$$
$$T_B - T_A - W_1 = 0$$
$$T_B = T_A + W_1$$
$$T_B = W_2 + W_1$$

Tension, Forces

Wednesday, January 10, 2007
7:59 AM

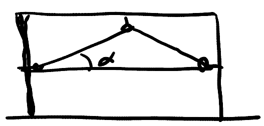


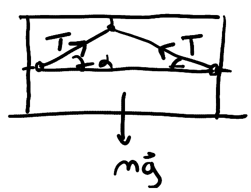
$A_x = 77\ kN$
$B = 400\ kN$
find A_y, C

$\Sigma F_x :\ A_x - C_x = 0$
$C_x = A_x$
$C_x = 77\ kN$

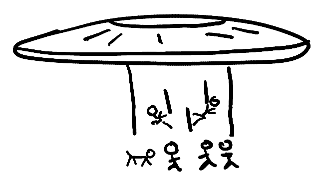
$\Sigma F_y :\ A_y - B + C_y = 0$
$A_y - 400 + \text{some \#} = 0$
$A_y = 400 - \text{some\#}$
$\boxed{A_y = 266.6\ kN}$

$C = \frac{C_x}{\sin 30}$
$\boxed{C = 154\ kN}$
$C_y = C \cos 30$
$C_y = \text{some \#}$

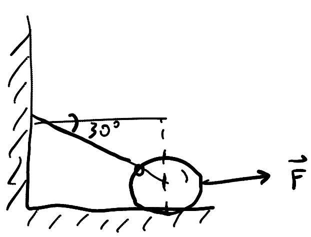




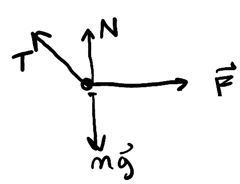
$m = 10\,kg$
$\alpha = 25°$
T?



$\Sigma F_x = 0$
$T\cos\alpha - T\cos\alpha = 0$ useless =tion
$\Sigma F_y = 0$
$2T\sin\alpha - mg = 0$
$T = \frac{mg}{2\sin\alpha} = 116.1\ N$



$\vec{F}$ 500N
surfaces are smooth, find $\vec{N}$ormal force
sphere weighs 50 kg



$\Sigma F_x = 0$
$\vec{F} - T\cos 30 = 0$
$\frac{\vec{F}}{\cos 30} = T$
$577.35N = T$

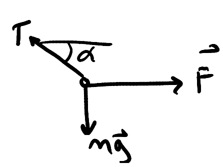
$\Sigma F_y = 0$
$\vec{N} - m\vec{g} + T\sin 30 = 0$
$\vec{N} = mg - T\sin\theta$
$\vec{N} = 201.8\ N$

Variation #1

Find α, T such that $N = 0$

$\Sigma F_y :\quad 0 = m\vec{g} - T\sin\alpha$
$T\sin\alpha = m\vec{g}$
$\frac{\vec{F}}{\quad} \sin\alpha = m\vec{a}$

$\Sigma F_x :\ \frac{\vec{F}}{\cos\alpha} = T$



$\cos\alpha$

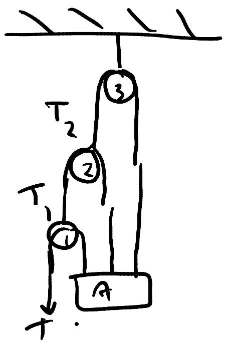
$F\tan\alpha = m\vec{g}$

$\tan^{-1}\frac{F}{mg} = \alpha$

$43.6^\circ = \alpha$

$T = \frac{mg}{\sin\alpha} = 700.4\text{ N}$

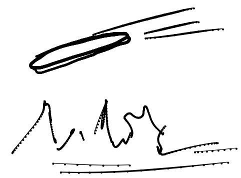
I WANT TO BELIEVE

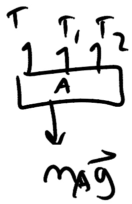


m_A = given

m_p = mass of pulleys given

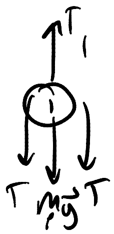
Find T





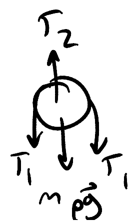
$\Sigma F_y = 0$

$T + T_1 + T_2 = m_A\vec{g}$



$\Sigma F_y = 0$

$T_1 = 2T + m_p\vec{g}$



$\Sigma F_y = 0$

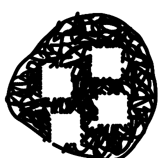
$T_2 = 2T_1 + m_p\vec{g}$

$T_2 = 2(2T + m_p\vec{g}) + m_p\vec{g} = 4T + 3m_p\vec{g}$

$T + 2T + m_p\vec{g} + 4T + 3m_p\vec{g} = m_A\vec{g}$

$7T + 4m_p\vec{g} = m_A\vec{g}$

$T = \frac{m_A\vec{g} - 4m_p\vec{g}}{7}$

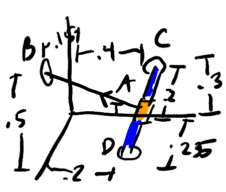


What is this???

Tension is a small fraction of weight of object we want to support.

Tension is reduced by weight of pulleys.

2.118



$T - 50\text{N}$

$\vec{T} = T \cdot \vec{e}_{AB}$

$\vec{r}_A = \vec{r}_C + \underbrace{\vec{r}_{CA}}_{.2(\hat{e}_{CD})}$

$\vec{T}_{\parallel} = \vec{T} \cdot \hat{e}_{CD}$

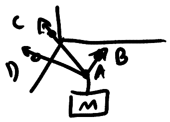
$\vec{T} = \vec{T}_{\parallel} + \vec{T}_{\perp}$

Forces, vectors, Tension

Friday, January 12, 2007

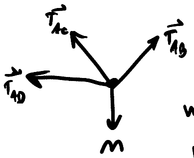
8:02 AM

HW #3 : 3.8, 3.27, 3.33, 3.57, 3.64, 3.82, 3.93 (M)



$m = 100\,kg$

A(0, −4, 0) m
B(4, 0, 2) m
C(−2, 0, −2) m
D(−3, 0, 3) m



we can characterize $\vec{T}_{A\_}$ as parameterizations of the points

$$\hat{e}_{AB} = (4\hat{\imath} + 4\hat{\jmath} + 2\hat{k})\left(\frac{1}{\underset{\text{magnitude}}{6}}\right) = \frac{2}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{1}{3}\hat{k}$$

$$\hat{e}_{AC} = (-2\hat{\imath} + 4\hat{\jmath} + -2\hat{k})\left(\frac{1}{\sqrt{24}}\right)$$

$$\hat{e}_{AD} = (-3\hat{\imath} + 4\hat{\jmath} + 3\hat{k})\left(\frac{1}{\sqrt{34}}\right)$$

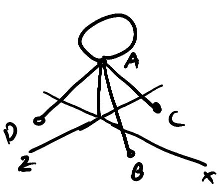
$$\Sigma F_x = 0 \;!\; \frac{2}{3}T_{AB} - \frac{2}{\sqrt{24}}T_{AC} - \frac{3}{\sqrt{34}}T_{AD} = 0$$

$$\Sigma F_y = 0 \;:\; \frac{2}{3}T_{AB} + \frac{4}{\sqrt{24}}T_{AC} + \frac{4}{\sqrt{34}}T_{AD} - mg = 0$$

$$\Sigma F_z = 0 \;:\; \frac{1}{3}T_{AB} - \frac{2}{\sqrt{24}}T_{AC} + \frac{3}{\sqrt{34}}T_{AD} = 0$$

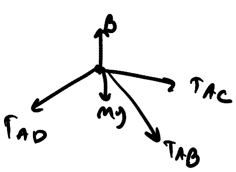
Solve

$T_{AB} = 519\ N \quad T_{AC} = 676\ N, \quad T_{AD} = 168\ N$



A(0, 8, 0)
B(16, 0, 16)
C(10, 0, −12)
D(−16, 0, 4)

$m = 90\,kg$
~~b~~ buoyancy = 1000 N



$$\hat{e}_{AB} = \frac{1}{24}(16\hat{\imath} - 8\hat{\jmath} + 16\hat{k})$$

$$\hat{e}_{AC} = \frac{1}{17.55}(10\hat{\imath} - 8\hat{\jmath} - 12\hat{k})$$

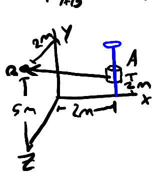
$$\hat{e}_{AD} = \frac{1}{18.33}(-16\hat{\imath} - 8\hat{\jmath} + 4\hat{k})$$

$\Sigma F_x :\ \frac{2}{3}T_{AB} + .57\,T_{AC} - .87\,T_{AD} = 0$

$\Sigma F_y :\ -\frac{1}{3}T_{AB} - .46\,T_{AC} - .44\,T_{AD} + \vec{B} - mg = 0$

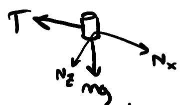
$\Sigma F_z :\ \frac{2}{3}T_{AB} - .68\,T_{AC} + .2\,T_{AD} = 0$

$T_{AB} = 64.8\,N,\ T_{AC} = 97.8\,N,\ T_{AD} = 114.6\,N$



200 kg slider, smooth bar held in place by cable. Tension in cable? Force exerted on slider?

z



$N_x + N_z = \vec{N}$

$\hat{e}_{AB} = \frac{1}{\sqrt{17}}(-2\hat{i} + (5-2)\hat{j} + 2\hat{k})$

$\Sigma F_x: -\frac{2}{\sqrt{17}}T + N_x = 0$ $\quad N = 1307\,N$

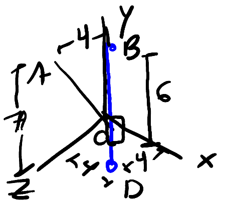
$\Sigma F_y: \frac{3}{\sqrt{17}}T - mg = 0$; $T = 2695\,N$

$\Sigma F_z: \frac{2}{\sqrt{17}}T + N_z = 0$; $N_z = -1307\,N$

Free Body Diagrams, vectors, forces

Tuesday, January 16, 2007

8:02 AM



A (0, 7, 4)

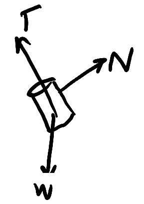
B (0, 7, 0)

C(   )

D(4, 0, 4)

m = 100 lb

$\vec{T}$, $\vec{N}$?



$\Sigma \vec{F} = \vec{0}$

$\vec{T} + \vec{N} - \vec{W} = \vec{0}$

$\vec{r}_C = \vec{r}_B + \vec{r}_{BC}$

$\hat{e}_{BC} = \hat{e}_{BD}$

$= \dfrac{\langle 4, -7, 4\rangle}{\sqrt{4^2+7^2+4^2}} = \langle \frac{4}{9}, -\frac{7}{9}, \frac{4}{9}\rangle$ $\quad \sqrt{81} = 9$

$\vec{r}_{BC} = 6\langle \frac{4}{9}, -\frac{7}{9}, \frac{4}{9}\rangle = \langle \frac{8}{3}, -\frac{14}{3}, \frac{8}{3}\rangle$

$\vec{r}_C = \vec{r}_B + \vec{r}_{BC}$

$\vec{r}_C = 7\hat{j} + \frac{8}{3}\hat{i} - \frac{14}{3}\hat{j} + \frac{8}{3}\hat{k} = \langle \frac{8}{3}, \frac{7}{3}, \frac{8}{3}\rangle$

$\hat{e}_{CA} = \frac{1}{5.54}(-\frac{8}{3}\hat{i} + \frac{14}{3}\hat{j} + \frac{4}{3}\hat{k})$

$\vec{T} = T\hat{e}_{CA}$

$\vec{N} \perp \hat{e}_{CB}$, so $\vec{N} \cdot \hat{e}_{CB} = 0$

$\vec{N} \cdot \hat{e}_{CB} = 0$

$\vec{N} = \vec{W} - \vec{T}$

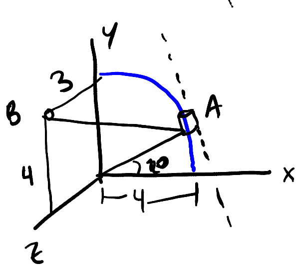
$(\vec{W} - \vec{T}) \cdot \hat{e}_{CB} = 0$

$\left[100\hat{j} - T\left(\frac{1}{5.54}(-\frac{8}{3}\hat{i} + \frac{14}{3}\hat{j} + \frac{4}{3}\hat{k})\right)\right] \cdot \langle \frac{4}{9}, -\frac{7}{9}, \frac{4}{9}\rangle = 0$

$(100\hat{j} + .48T\hat{i} - .84T\hat{j} - .24T\hat{k}) \cdot \langle \frac{4}{9}, -\frac{7}{9}, \frac{4}{9}\rangle = 0$

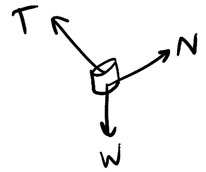
$T = 102.88$ lbs

$\vec{N} = (49.4\hat{i} + 13.6\hat{j} - 24.7\hat{k})$ lb



w = 100 lbs

r = 4



$\vec{T} + \vec{N} - \vec{W} = \vec{0}$

$\langle 4\cos 20,\ 4\sin 20,\ 0 \rangle = A$
$\langle 0, 4, 3 \rangle = B$

$\hat{e}_{AB} = -.686\hat{\imath} + .480\hat{\jmath} + .547\hat{k}$

$\vec{T} = T\,\hat{e}_{AB}$

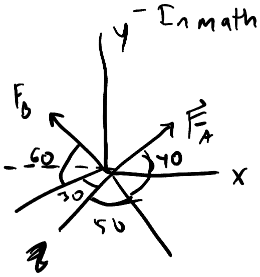
$\vec{r}_A = 4\cos 20\,\hat{\imath} + 4\sin 20\,\hat{\jmath}$
$\hat{e}_A = \cos 20\,\hat{\imath} + \sin 20\,\hat{\jmath}$
$\hat{e}_{\text{bar axis}} = -\sin 20\,\hat{\imath} + \cos 20\,\hat{\jmath}$ derivative of $\hat{e}_A$

$\vec{N} \cdot \hat{e}_{\text{bar axis}} = \vec{0}$
$(\vec{w} - \vec{T}) \cdot \langle -\sin 20, \cos 20 \rangle = 0$
solve for T

$T = 137$ lbs
$\vec{N} = 94\hat{\imath} + 34\hat{\jmath} - 75\hat{k}$ lbs

I WISH I WERE...

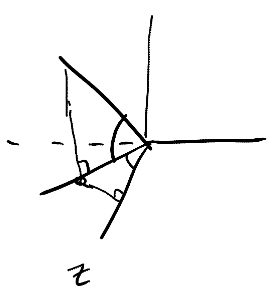
- In an F/A 22 or F-35
- Watching X-files
- <u>Anyplace</u> else
- Writing my book
- Playing hockey
- Playing video games
- Graduated
- Scully / Mulder
- Shooting ducks (Duckhunt)
- At home
- Eating breakfast
- In math class
- In a space shuttle
- An FBI agent
- <u>NOT</u> bored!
- Going to the moon, ~~Mars~~, or ISS
- Playing in the snow



$|\vec{F}_B| = 400$ N
$|\vec{F}_A + \vec{F}_B| = 900$ N   Find $|\vec{F}_A|$

b/c ∡ opposite to x axis

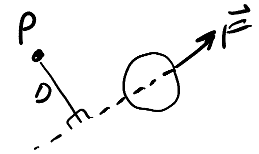
$\vec{F}_B = (F_B \cos 60)\sin 30\,\hat{\imath} + (F_B \sin 60)\cos 30\,\hat{\jmath}$
$\vec{F}_A = (F_A \cos 40)\sin 50\,\hat{\imath} + (F_A \sin 40)\hat{\jmath} + (F_A \cos 40)\cos 50\,\hat{k}$



Moments

A moment exerted by a force about a point is the product of force × the perpendicular distance from point to the line of action of force.

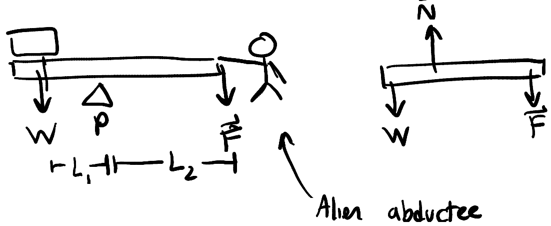
A moment describes the rotation a force induces.



$$\vec{M}_{FP} = |\vec{F}| D$$
$$= D|\vec{F}| \quad N\text{-}m$$

(+)

Objects lying on a plane



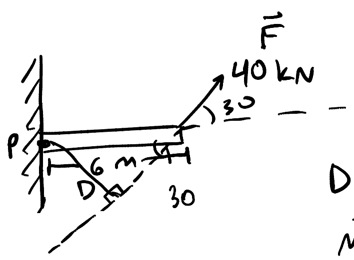
$$\Sigma \vec{F}_y = \vec{0}$$
$$\vec{N} - \vec{F} - \vec{W} = 0$$
$$\Sigma \vec{M}_P = 0$$
$$-L_2 F + 0(N) + L_1 W = 0$$

(+) b/c bar moves in ↶ direction

$$L_1 W = L_2 F$$
$$F = \frac{L_1 W}{L_2}$$

So the external force will be much smaller than what it takes to not let the object topple. $N - W = \frac{L_1 W}{L_2}$

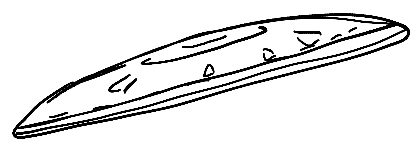
$$N = \frac{L_1 W}{L_2} + W$$



$$D = 6(\sin 30) = 3$$
$$\vec{M}_{FP} = 3\,m\,(40\,kN)$$
$$\vec{M}_{FP} = D \cdot F = 120000\ N\text{-}m$$

OR

$$F_x = 40 \cos 30$$
$$F_y = 40 \sin 30$$
$$M_{F_xP} = 0 \qquad M_{F_yP} = 6 \cdot 40kN \sin 30$$
$$M_{F_yP} = 120{,}000\ N\text{-}m$$

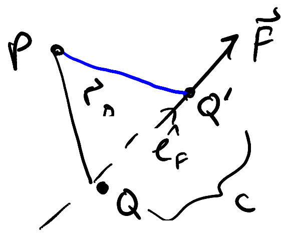


FBI
FEDERAL BUREAU OF INVESTIGATION

This is like the cross product.

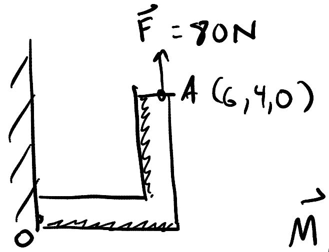
This is like the cross product

$\vec{M}_{Fp} = \vec{r} \times \vec{F}$ where $\vec{r}$ is a position vector for any point on the line of action of F w.r.t. P



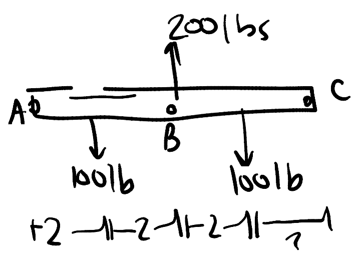
$$\vec{r}_{Q'} = \vec{r}_Q + C\hat{e}_F$$

$$\vec{r}_Q \times \vec{F} = \vec{r}_{Q'} \times \vec{F}$$
$$= (\vec{r}_Q + C\hat{e}_F) \times \vec{F}$$
$$= \vec{r}_Q \times \vec{F} + \underbrace{C\hat{e}_F \times \vec{F}}_{\text{this is 0 b/c } \sin 0 = 0}$$

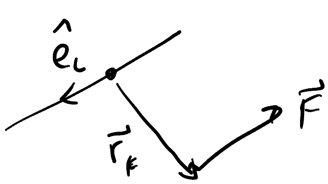


$$\vec{M}_{Fo} = \vec{r}_A \times \vec{F}$$
$$= (6\hat{i} + 4\hat{j}) \times 80\hat{j}$$
$$= 480\,\hat{k} \quad \text{N-m}$$

CSI
CRIME SCENE INVESTIGATION



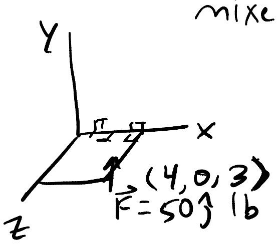
$$\Sigma M_A = -200 + 800 - 600 = 0 \text{ ft-lb}$$
$$\Sigma M_B = 0$$
$$\Sigma M_C = 0$$



$$\vec{M}_P = \vec{r}_F \times \vec{F}$$

$$(\hat{e}_L \cdot \vec{M}_{Fp})\,\hat{e}_L$$

$$\underbrace{(\hat{e}_L \cdot (\vec{r}_F \times \vec{F})}_{\text{mixed triple product}})\,\hat{e}_L$$



$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 4 & 0 & 3 \\ 0 & 50 & 0 \end{vmatrix} \hat{i}$$

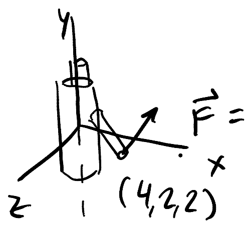
$$1(-150)\hat{i} + 0(\text{blah}) + 0(\text{blah}) = \vec{M}$$

Moments, Couples

Friday, January 19, 2007
8:03 AM

HW #4 Ch.4 8, 18, 63, 93, 100, 176 (M)
Exam in 2 weeks, Thurs Feb 1st (Handout on Blackboard)

Practice exams on Blackboard, mockup exam next week (50 min)



$\vec{F} = 100\hat{\imath} + 60\hat{\jmath} - 40\hat{k}$

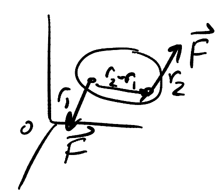
$$\vec{M}_y = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 0 \\ 4 & 2 & 2 \\ 100 & 60 & -40 \end{vmatrix} \hat{\jmath} \qquad 0\hat{\imath} - |\text{stuff}|\hat{\jmath} + 0\hat{k}$$

$$-1(-160 - 200)\hat{\jmath} = 360\hat{\jmath} \text{ lb-ft}$$

I know a song that gets
on everybody's nerves...
everybody's nerves...
everybody's nerves!

Couples: - A force exerts no net force on an object but does exact a moment
- Forces are equal in alignment, opposite in direction & have different lines of action.

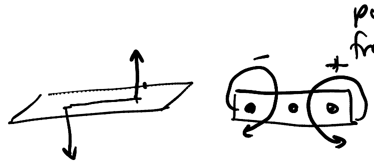
Object spins / rotates



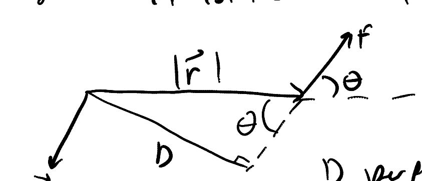
Sum of moments about O:

$$\Sigma\vec{M}_O = \vec{r}_1 \times (-\vec{F}) + \vec{r}_2 \times (\vec{F})$$
$$= (\vec{r}_2 - \vec{r}_1) \times \vec{F}$$
$$= \vec{r} \times \vec{F}$$

The distance between the 2 points is all that matters. Distance from origin doesn't matter.

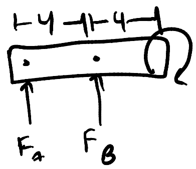


$|\vec{M}| = |\vec{r}||\vec{F}|\sin\theta \qquad |\vec{r}|\sin\theta = D$



D perpendicular distance between lines of action of forces

D is the smallest distance between lines of action of forces.



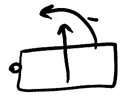
$M = 200$ ft-lb

$\Sigma F_y = 0$: $F_A + F_B = 0$

$\Sigma \vec{M}_A = 0$ $4 F_B - 200 = 0$

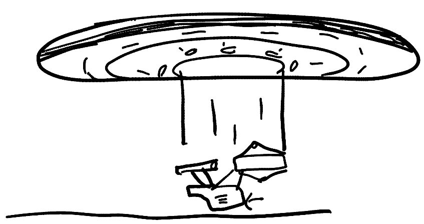
$F_B = 50$ lb

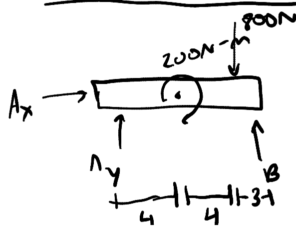
$F_A = -50$ lb



so F_B ↺

The position doesn't matter when describing the couple, but it does matter in describing the movement itself





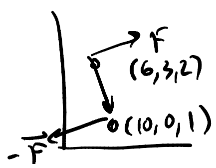
$\Sigma \vec{F}_x = 0$ $A_x = 0$

$\Sigma \vec{F}_y = 0$ $A_y + B - 800 = 0$

$\Sigma M_A = 0$ $200 - 8(800) + 11 B = 0$

$B = 600$ N

$A_y = 200$ N



$\vec{F} = 40\hat{i} + 24\hat{j} + 12\hat{k}$ N

Find shortest distance between lines of action

$$|\vec{M}_c| = |\vec{F}| \underbrace{|\vec{r}| \sin\theta}_{D}$$

$$\vec{M}_c = \vec{r} \times \vec{F} \qquad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & -1 \\ 40 & 24 & 12 \end{vmatrix}$$

Equivalent Systems

Monday, January 22, 2007
8:00 AM

Equivalent Systems

System of Forces and Moments ≡ set of forces of moments and couples

2 systems of this kind are equivalent (≡) if these conditions hold:

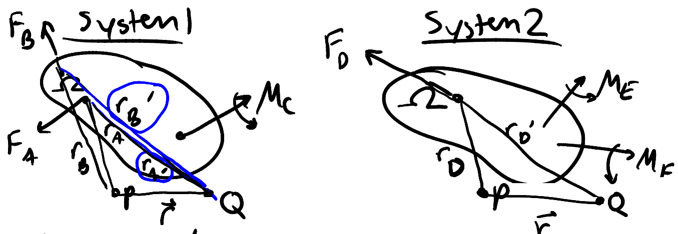
- ΣF in system 1 = ΣF in system 2
- ΣM in system = ΣM in system 2 about the same pt. P

Point P always exists in system 2 (we're just condensing & containing the forces in the system).

If these conditions hold, then ΣM_Q in system 1 = ΣM_Q in system 2 for <u>any</u> point Q.

So now to prove this:

Consider these systems



r_A', r_B, r_B', r_A are related through $\vec{r}$

$r + r_A = r_A'$ $\quad r + r_B = r_B'$ $\quad r + r_D = r_D'$ $\quad (\Sigma M_Q)_1 = (\Sigma M_Q)_2$

$$r_A' \times F_A + r_B' \times F_B + M_C = r_D' \times F_D + M_E + M_F$$

$$(r+r_A)\times F_A + (r+r_B)\times F_B + M_C = (r+r_D)\times F_D + M_E + M_F$$

$$\underbrace{r_A\times F_A + r_B \times F_B + M_C}_{1} + r\times(F_A + F_B) = \underbrace{r_D\times F_D + M_E + M_F}_{2} + r\times F_D$$

1 = 2

Σ Moments about point P = 1 in system 1

Σ Moments about point P = 2 in system 2

Cancel out b/c $(\Sigma M_P)_1 = (\Sigma M_P)_2$

$$r\times\underbrace{(F_A + F_B)}_{3} = r\times \underbrace{F_D}_{4}$$

3 = 4

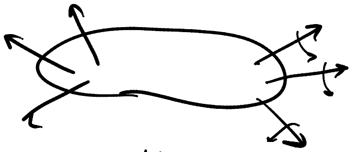
Σ Forces about point P = 3 in system 1

Σ Forces about point P = 4 in system 2

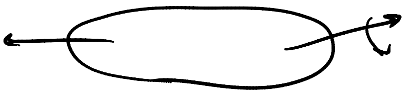
$(\Sigma F_P)_1 = (\Sigma F_P)_2$

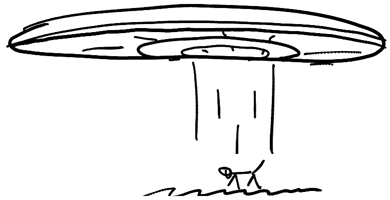
Regardless of how complicated a system is, it can <u>always</u> be represented by a single force at a given point and a single represented by a single force at a given point and a single couple. i.e. $F = (\Sigma F)_1$; $M = (\Sigma M_P)_1$

where $F = (\Sigma F)_2$ $M = (\Sigma M_P)_2$

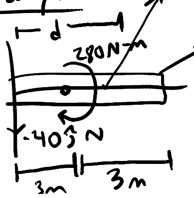


||





Example



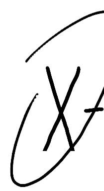
Determine where Line of action F intersects x axis

$F = (\Sigma F)_1 = 60\hat{i} + 20\hat{j}$ N

$M = (\Sigma M)_1 = -280 + \overset{360}{6(60)} = 80$ distance = 6m

$\therefore 20d = 80$

$d = 4$ m



Objects in equilibrium

$\Sigma F = 0$

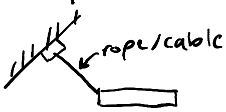
$\Sigma M = 0$

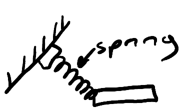
System of forces + moments acting on an object in equilibrium is equivalent to a system of no forces and no couples.

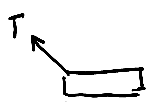
Applications (2D)

supports represented by stylized modeles called support connections

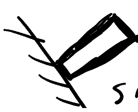
Support | Reactions





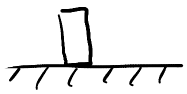


Colinear force, (||) w.r.t. cable/ spring

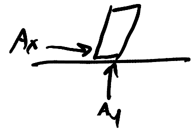


smooth contact (no friction)

A is Normal (⊥) to surface

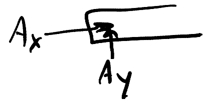


rough contact

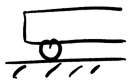


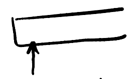
Friction component A_x

Pin (frictionless) support

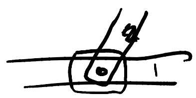


2 Force components



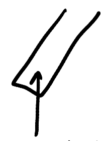


A ⊥ to supporting surface

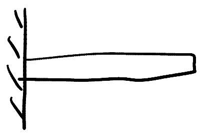


Collar connects to bar 2 smooth contact

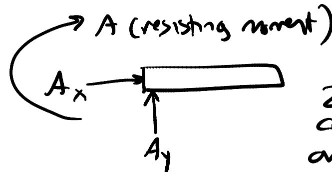
Constrained slider



A ⊥ to constraint (bar 1)



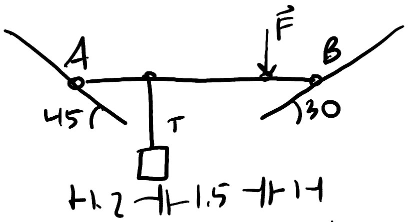
Fixed support



2 force components and 1 moment

Tension on Bar

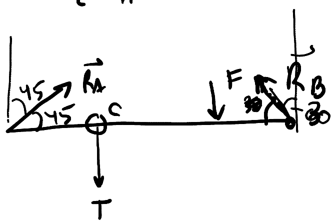
Wednesday, January 24, 2007
8:03 AM



$\vec{F} = 400$

Find $\vec{R}_A, \vec{R}_B$

Bar is weightless



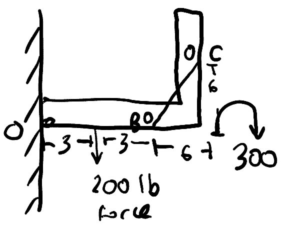
$\Sigma F_x = 0 = \vec{R}_A \sin 45 = \vec{R}_B \sin 30$

$\Sigma F_y = 0 = \vec{R}_A \cos 45 + \vec{R}_B \cos 30 - T - F$

$\Sigma M_C = 0 = -(\vec{R}_A \cos 45)1.2 - 1.5\vec{F} + (R_B \cos 30)2.5$

$R_A = 271.1\ N$

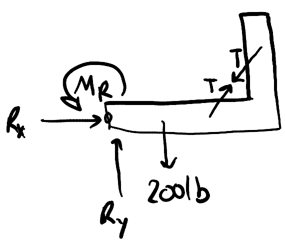
$R_B = 383.4\ N$



fixed support

$T_{BC} = 100\ lb$

Determine reactions at fixed support, O



Because T's are = and opposite, no moment. There is also no perpendicular distance between them (b/c they coincide) and therefore there is no couple (no moment results)

T is an internal force, as well, and free body diagrams are only for external forces.

$\Sigma F_x = 0 \qquad R_x = 0$

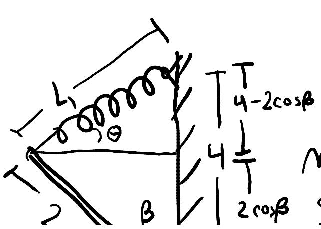
$\Sigma F_y = 0 \qquad R_y = 200\ lb$

$\Sigma M_O = 0$

$M_R - 600 - 300 = 0 \qquad \underbrace{(200\ lb)(3m)}_{=600}$

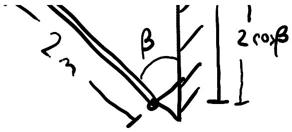
$M_R = 900\ ft\text{-}lbs$

This means counter clockwise b/c positive.



$m_{bar} = 36\ kg$

Spring unstretched when $\beta = 0$



Spring unstretched when $\beta = 0$
System in equilibrium when $\beta = 30°$
length of unstretched spring = 2 m (L_0)
k?

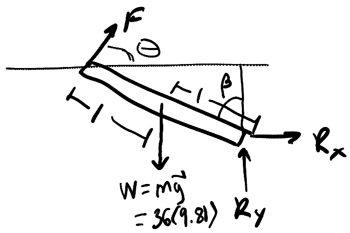
$$k = \frac{F}{(L_1 - L_0)}$$

Law of Cosines

$$L_1^2 = 16 + 4 - 2 \cdot 8\cos\beta$$

$$L_1 = 2.48 \text{ m}$$

$$\Theta = 66.25°$$ b/c $$\sin\Theta = \frac{4 - 2\cos\beta}{L_1}$$



no moment reaction

weight is a body force, but we can assume it's in the c.m.

$$\Sigma M_R = 0$$

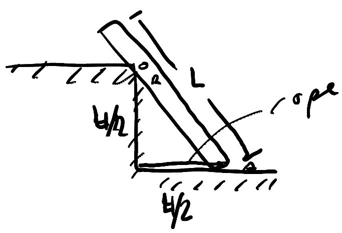
$$-(F\cos\Theta)(2\cos\beta) - (F\sin\Theta)(2\sin\beta) + mg(\sin\beta) = 0$$

solve for F, only unknown.

$F = 109.5$ N

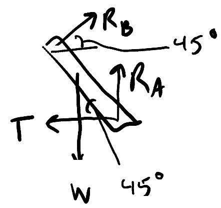
Then solve for R = 2281 N/m

HW: 4.116, 4.147, 5.44, 5.61, 5.75, 5.88



weight W

T ?



$\Sigma F_x = 0$

$R_B \cos 45 = T$

$\Sigma F_y = 0$

$R_B \sin 45 + R_A - W = 0$

$R_B \sin 45 + R_A = W$

$\Sigma M_A = 0$

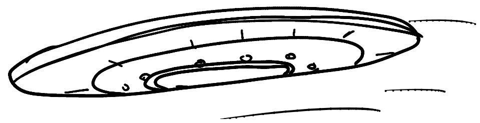
$\left(\frac{L}{2}\right)(\cos 45)\,W - \sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2}\; R_B = 0$

$\frac{L}{2}\,\frac{\sqrt{2}}{2}\,W = \frac{\sqrt{2}}{2}\,L\,R_B$

$\frac{W}{2} = R_B$

$T = \frac{W}{2}\cos 45$

$T = \frac{\sqrt{2}}{4}\cos 45$



Static Indeterminacy

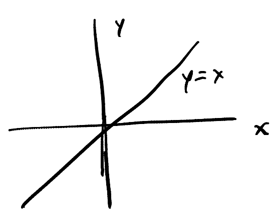
Improperly supported systems

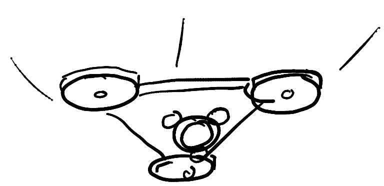
solving systems of equations

$x + y = 2$
$x - y = 0$ } $x = 1,\ y = 1$

This is a unique soln b/c the system is linear. They are linearly independent. Variables are degrees of freedom.

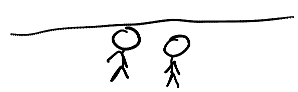
$y = x$

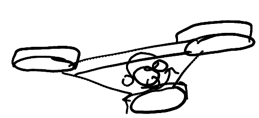


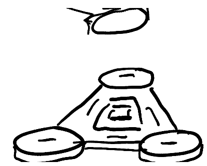


You end up w/ an infinite # of solutions b/c you cannot uniquely determine or identify the system that we are interested in.

Static Indeterminacy





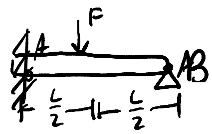


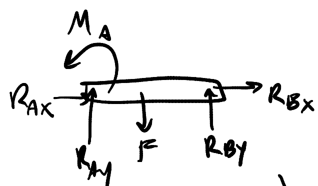
2 variables
1 equation

2 variables → equations redundant (linear combinations)
3 equations → inconsistent equations (no solution)

$x+y=3$ ← restricts to different point (impossible to be at both points simultaneously, and so system is inconsistent)
$x+y=2$ ← restricts motion to 1 point
$x-y=0$ ← gives you $x=y$

Static indeterminacy arises when we have redundant supports



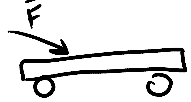


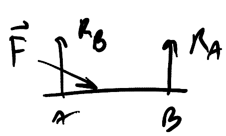
$\Sigma F_x=0$
$R_{Ax}+R_{Bx}=0$
$\Sigma F_y=0$
$R_{Ay}+R_{By}-F=0$
$\Sigma M_A=0$
$-(\frac{L}{2})F+(L)R_{By}+M_A=0$
need more eqns, so:
$\Sigma M_B=0$
$-(L)R_{Ay}+(\frac{L}{2})F+M_A=0$
$(\Sigma M_A=0)-(\Sigma M_B=0)$
$-LF+LR_{By}+LR_{Ay}=0$
$-F+R_{By}+R_{Ay}=0$ Redundant

X FILES

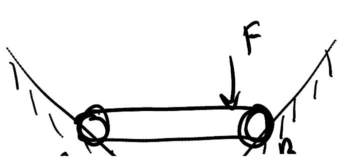
I WANT TO BELIEVE

Improper Supports
System is not in equilibrium

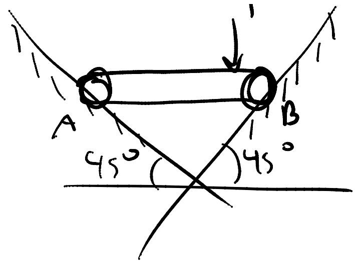




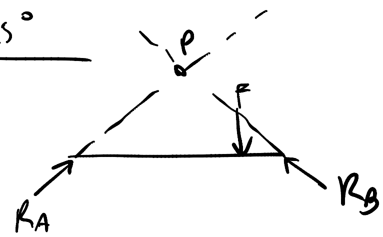
System will move



System will move

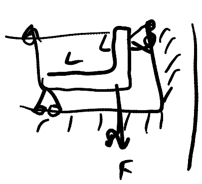


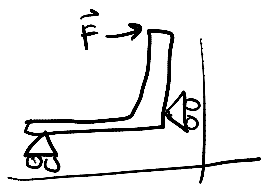
System will move

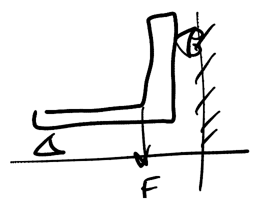


Listen up and you'll hear a
tale, a tale of a fateful trip
That started on this tropic
shore, aboard this tiny ship.

3 Force System. If you extend lines of action, they intersect at point P. This means they're concurrent. Because they intersect, M = 0 and they will cancel each other out. This means the bar will rotate.







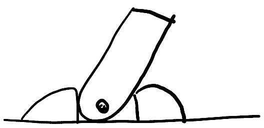
System is supported

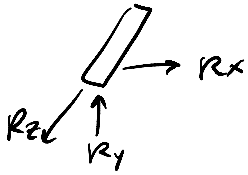
3D Supports (And the Mulder Abduction Mytharc)

Monday, January 29, 2007
8:04 AM

3D supports   Table 5-2

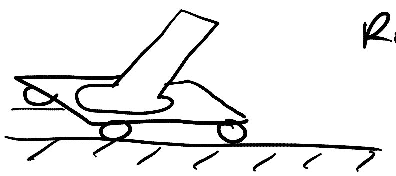
Ball-in-socket support.

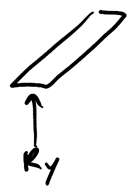




no moments

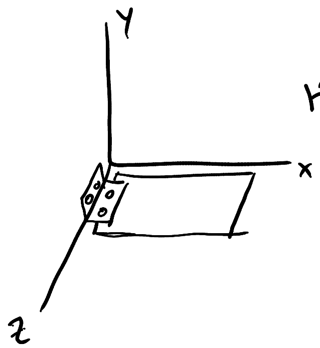
Roller support

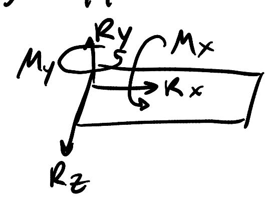




no moments

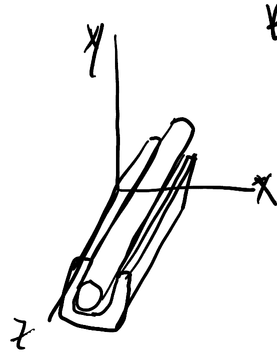
Hinge support

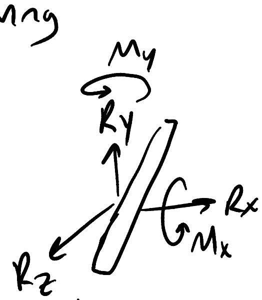




Mx and My

Bearing

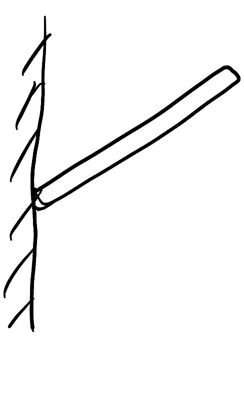


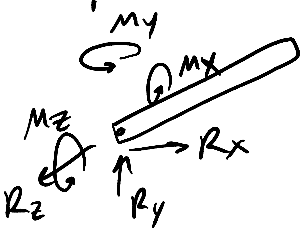


Mx & My

Restricts motion in x & y.

Fixed support

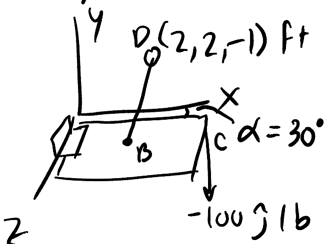




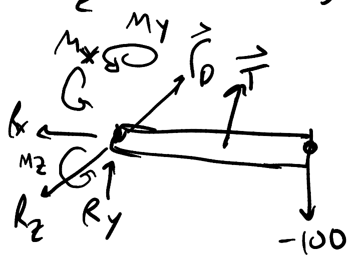
Mx, My, & Mz.

Restricts motion in x, y, z

Hinge

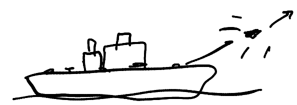


B is mid pt of bar
Bar is 4' long
Bar subjected to force at C = -100 lbs
Find reaction at hinge.



$B(2\cos 30, -2\sin 30, 0)$

$\hat{e}_{BD} = .084\hat{i} + .945\hat{j} - .315\hat{k}$ ft



$\Sigma F_x = 0$

$R_x + .084|\vec{T}| = 0$

$\Sigma F_y = 0$

$R_y + .945|\vec{T}| - 100 = 0$

$\Sigma F_z = 0$

$R_z + (-.315)|\vec{T}| = 0$

Moments:

$$\vec{r}_D \times \hat{e}_{BD} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ .084T & .945T & -.315T \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4\cos 30 & 0 & 0 \\ 0 & -100 & 0 \end{vmatrix} + M_x\hat{i} + M_y\hat{j} = 0$$

$.315T\hat{i} + .546T\hat{j} + 1.722T\hat{k} - 346\hat{k} + M_x\hat{i} + M_y\hat{j} = 0$

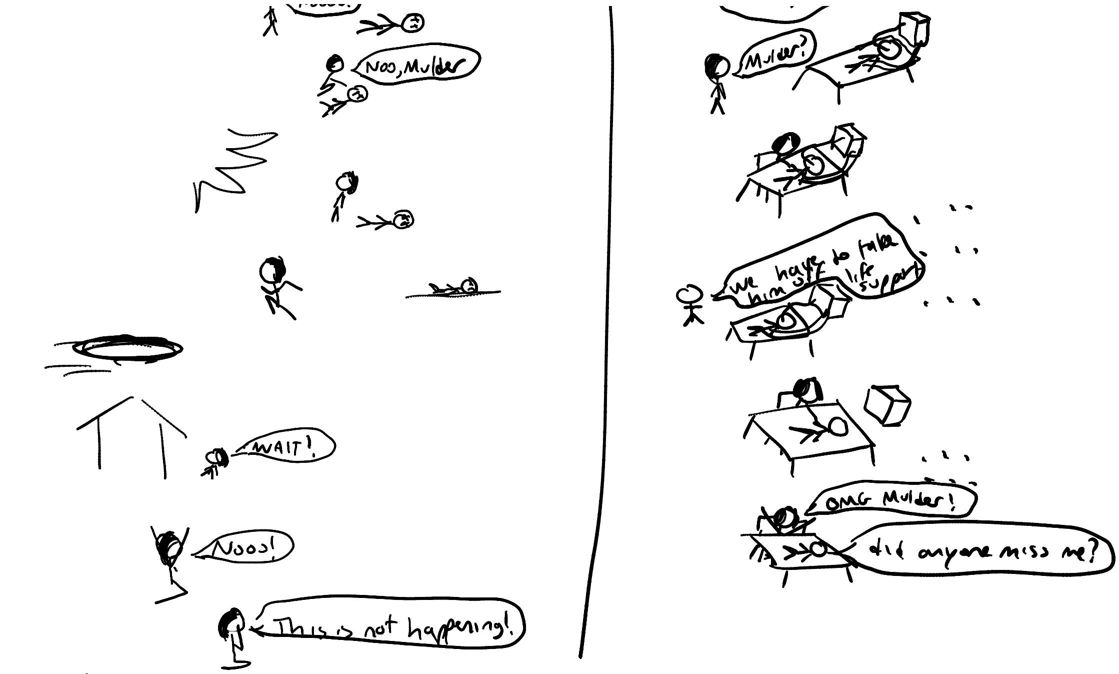
$.315T + M_x = 0$ $\quad \Sigma M_x$

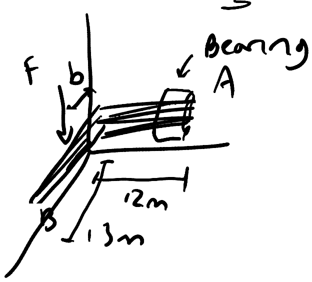
$.546T + M_y = 0$ $\quad \Sigma M_y$

$1.722T - 346 = 0$ $\quad \Sigma M_z$

$1.722T = 346$

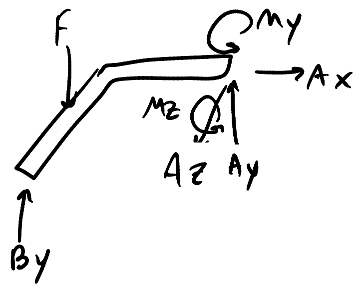
$T = 201$ lbs then get M_x & M_y





$b = .15\ m$

$F = 4\ kN$



$\Sigma F_x = 0$

$A_x = 0$

$\Sigma F_y = 0$

$B_y - F + A_y = 0$

$\Sigma F_z = 0$

$A_z = 0$

$\Sigma \vec{M}_A = \vec{0}$

$$\vec{r}_{FA} \times \vec{F} \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -.2 & 0 & .15 \\ 0 & -4 & 0 \end{vmatrix} + \vec{r}_{BA} \times \vec{B_y} \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -.2 & 0 & .3 \\ 0 & B_y & 0 \end{vmatrix} + M_y\hat{\jmath} + M_z\hat{k} = \vec{0}$$

$$.6\hat{\imath} + .8\hat{k} - .3B_y\hat{\imath} - .2B_y\hat{k} + M_y\hat{\jmath} + M_z\hat{k} = \vec{0}$$

$.6 - .3B_y = 0$

$M_y\hat{\jmath} = 0$

$.8 - .2B_y + M_z = 0$

Solve, plug in

$A_x = 0$ $\qquad M_y = 0$

$A_y = 2$ $\qquad M_z = -.4\ kN$

$A_y = ?$ $M_z = -.4\ kN$

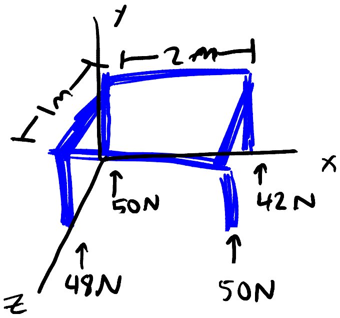
$A_z = 0$

$B_y = 2\ kN$

Multiple Force Systems

Tuesday, January 30, 2007

8:02 AM



Midterm

Equivalent System Problem

3D problem (equilib)

Springs, pulleys

Replace w/ equivalent system w/ one force & one couple acting at origin.

$(\Sigma F) = F_o$

$F_o = 190\,N$

$\Sigma M_O = M_O$

F_o intersects O, and therefore isn't counted in this

$\vec{M} = \vec{r}_{of} \times \vec{F}$

Moments are vectors so we need to attach

$M_o = -48(1)\hat{\imath} - 50(1)\hat{\imath} + 50(2)\hat{k} + 42(2)\hat{k}$

counter clockwise rotation about x axis.

distance from axis of rotation

$M_o = -98\,\hat{\imath} + 184\,\hat{k} \quad N\cdot m$

Find point of application of force

$\Sigma\vec{F} = F_{xz}$

$190\,N = F_{xz}$

$\Sigma\vec{M}_o = \vec{M}_{of_{xz}}$

$$-98\hat{\imath} + 184\hat{k}\ N\cdot m = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & 0 & z \\ 0 & 190 & 0 \end{vmatrix}$$

$-98\hat{\imath} + 184\hat{k} = \hat{\imath}(190z) + \hat{k}(190x)$

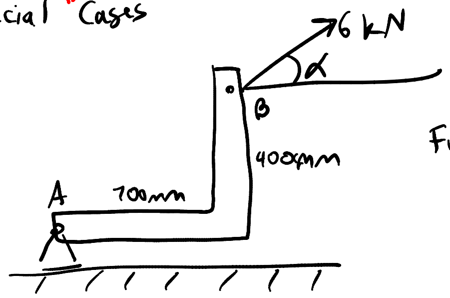
$-98\hat{\imath} = -190z\hat{\imath}$

$= z$

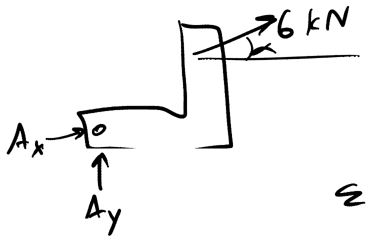
$184\hat{k} = 190x\hat{k}$

$= x$

Special [AGENT] Cases



Find α, A_x, A_y



Free body diagram

$\Sigma F_x = 0$

$A_x + 6\cos\alpha = 0$

$\Sigma F_y = 0$

$A_y + 6\sin\alpha = 0$

$\Sigma M_A = 0$

$-6\cos\alpha \cdot (.4m) + 6\sin\alpha(.7m) = 0$

$6\sin\alpha(.7) = 6\cos\alpha(.4)$

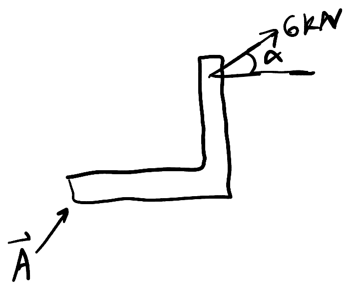
$\tan\alpha = \frac{.4}{.7}$

$\tan^{-1}\left(\frac{.4}{.7}\right) = \alpha$

$29.75° = \alpha$

$A_x = -5.21\ N$

$A_y = -2.98\ N$

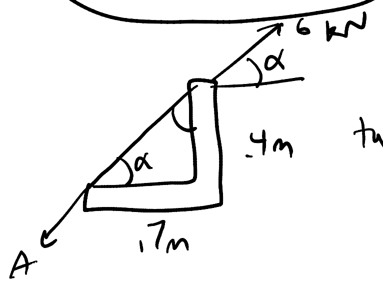


equal in magnitude
opposite in direction, since bar is in equilib

2 forces = in mag & opp direction are couples, which produce moments. The Moment will be magnitude of 2 forces x distance between lines of action of forces.

$D \cdot |\vec{F}| = 0$ ← equilib

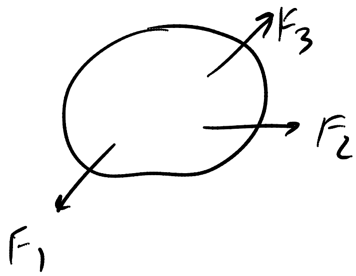
cannot be 0, so is 0.

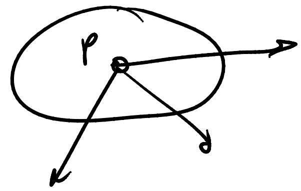


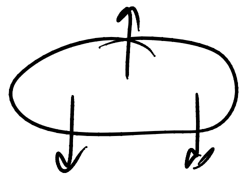
$\tan\alpha = \frac{4}{7}$

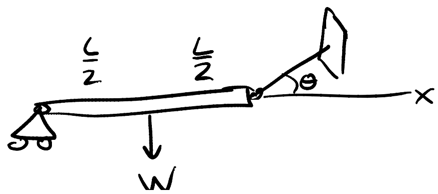
3 Force system

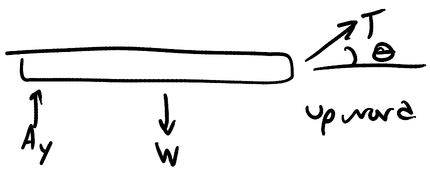
If object is in equilib, 1) Forces are coplanar
2) forces are either ∥ or concurrent



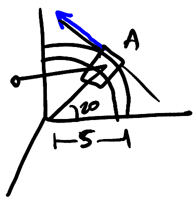








If bar is in equilib, $\theta = 90$ b/c there needs to be an upward force.



Bar axis is line tangent to bar at A.

$\vec{r}_A = 5\cos 20\,\hat{i} + 5\sin 20\,\hat{j} + 0\,\hat{k}$

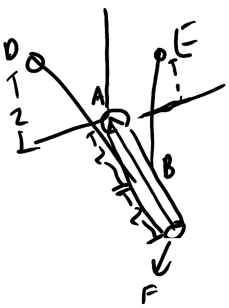
tangent is $\perp$ to $\vec{r}_A$, and we can take the derivative of each pt.

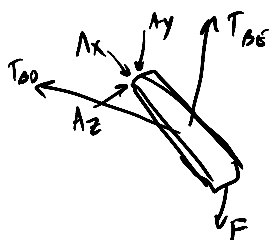
$\hat{e}_A = \cos 20\,\hat{i} + \sin 20\,\hat{j}$

$\hat{e}_A' = -\sin 20\,\hat{i} + \cos 20\,\hat{j}$

to the left   upward

Dot product between $\hat{e}_A'$ and $\vec{r}_A = 0$ b/c $\perp$.





functions

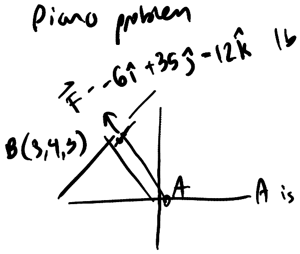
$T_{BD}(F) \le 25$

$T_{BE}(F) \le 25$

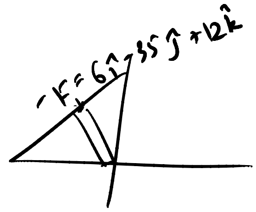
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ .624T_{BD} & .624T_{BD} & -.468T_{BD} \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ \frac{2}{3}T_{BE} & \frac{1}{3}T_{BE} & \frac{2}{3}T_{BE} \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 0 \\ 0 & -F & 0 \end{vmatrix} = \vec{0}$$

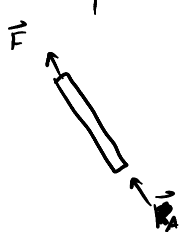
$T_{BE} \le 25$ kN $T_{BD} \le 25$ kN

Piano problem



A is on xz plane $A_y = 0$





$\vec{R}_A = \vec{F}$

$\hat{e}_{BA} = \frac{1}{|\vec{F}|}(\vec{F}) = .16\,\hat{i} - .93\,\hat{j} + .32\,\hat{k}$

$\vec{r}_A = \vec{r}_B + \ell\,\hat{e}_{BA}$ $\boxed{\triangle \text{ rule of vector addition}}$

$r_{Ay} = 0 = 4 + \ell(-.93)$

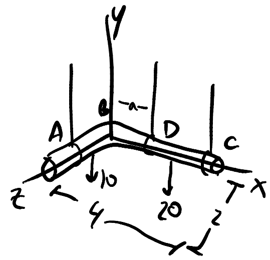
$4.3 = \ell$ ft

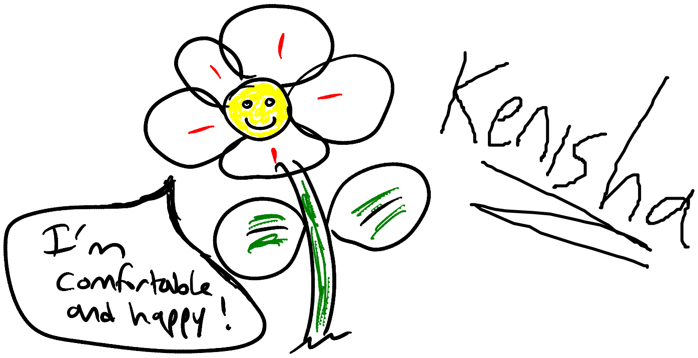
Sample Force Moment Prob

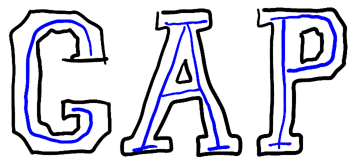
Friday, February 02, 2007

8:11 AM

1.







$\Sigma F_y = 0 = A + C + D = 30\text{ lb}$

$\Sigma M_x = 0 = -2A + 10 = 0$

$A = 5\text{ lb}$

$\Sigma M_z = 0 = aD + 4C - 40 = 0$

$aD + 100 - 4D - 40 = 0$

$D = \frac{60}{4-a}$

$C = 3.18\text{ lb} \qquad D = 28.18\text{ lb}$

As you move rope D to the right, you expect the structure to rotate.

A, C, D need to be non-negative or pointing upward ≥ 0

$A = 5\text{ lb} \qquad C, D \geq 0$

$\frac{60}{4-a} \geq 0$

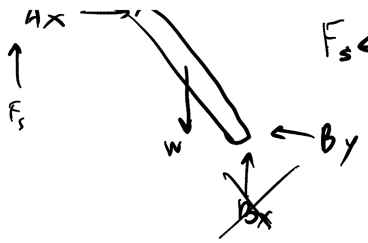
$a \leq 4$ and that makes sense b/c the bar is only 4 ft long.

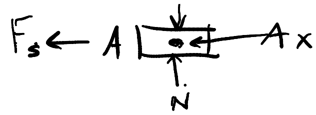
$C \geq 0$

$25 - \frac{60}{4-a} \geq 0$ substituting from earlier

$4 - a \geq \frac{60}{25}$

$a \leq 4 - \frac{60}{25} = \boxed{1.6\text{ feet}}$





$A_x = -F_s$

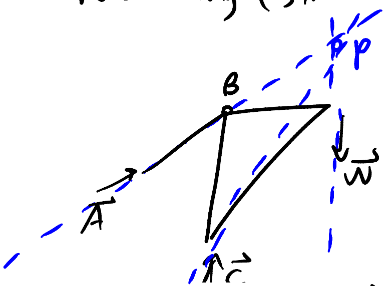
$A_y = N$

$\underset{\text{displacement of spring}}{D} = \ell - \ell\cos\theta$

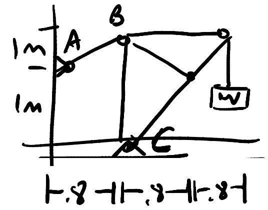
Trusses

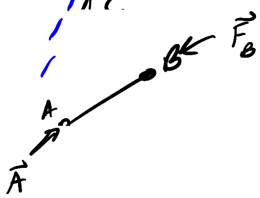
Monday, February 05, 2007
8:04 AM

Truss thing (structure that supports weight)



3 force system





2 force member

Structure is an object that is capable of supporting or exerting loads.

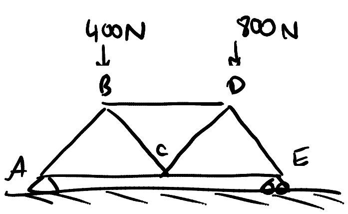
- Composed of interconnected members
- Understand both forces & moments on whole structure as well as on individual members

Structures

Trusses - Structures that are comprised of 2 force members. Loads are applied on the joints

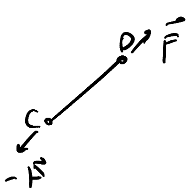
Frames - structures that are designed to remain stationary

Machines - structures designed to move

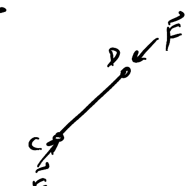


Warren Truss

→ Members have same length

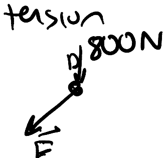


Case of tension



Case of compression

tension



pos. F b/c diff free body diagram

we're talking about the pin, & the pin is pulled inward while the bar is pulled outward.

compression

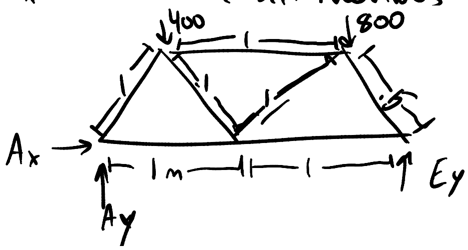
not changing the sign b/c diff free body diagrams

tensile forces are positive

pulled outward.

tensile forces are positive
Compressive forces are negative

- Reactions
- Axial forces on all members



$\Sigma F_x = 0$

$A_x = 0$

$\Sigma F_y = 0$

$A_y + E_y = 1200$

$\Sigma M_A = 0$

$-.5(400) - 1.5(800) + 2E_y = 0$

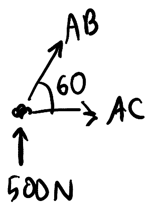
$E_y = 700$

$A_y = 500$

So those are the rxns. Now we find axial forces
Draw free body diagrams of joints.

Joint A

60° b/c equilateral Δ.



$\Sigma F_x = 0$

$AC + AB\cos 60 = 0$

$\Sigma F_y = 0$

$500 + AB\sin 60 = 0$

$AB = -577.35$ N

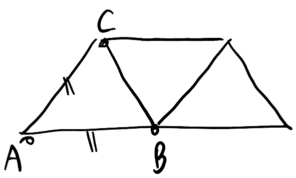
so AB points opposite of the way we drew it.

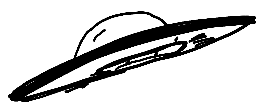
this means compression! (−)

$AC + (-577.35)\cos 60 = 0$

$AC = 288.68$ N

this means tension b/c (+)





|| = found axial forces.
So C is the best candidate + proceed. Only 2 unknowns vs. B's 3 unknowns.

Joints

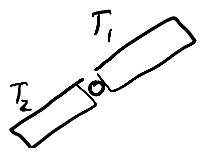
Tuesday, February 06, 2007

8:00 AM

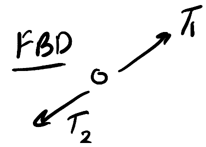
Joints w/out external loads

Special Cases

1) Case of a joint that connects 2 collinear members



T_1, T_2 = axial forces

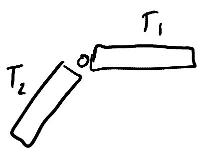


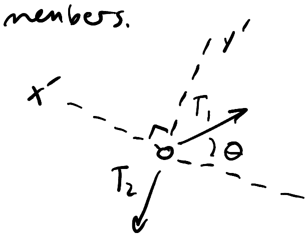
$T_1 = -T_2$

$T_1 = T_2$ if you ignore direction

Both in tension.

2) Case of 2 noncollinear members.





$\Sigma F_{x'} = 0$

$T_1 \cos\theta = 0$

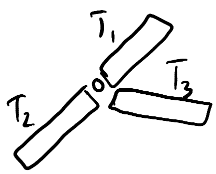
$\boxed{T_1 = 0}$

$\Sigma F_{y'} = 0$

$T_1 \sin\theta = T_2$

$\boxed{0 = T_2}$

3) Case of 3 members; 2 collinear, one noncollinear.

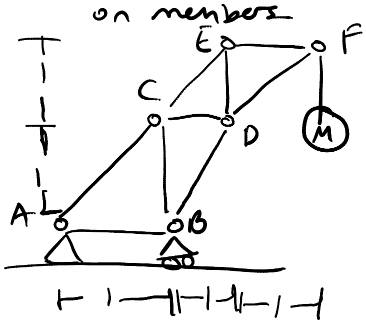


$T_3 = 0$

$T_1 = T_2$

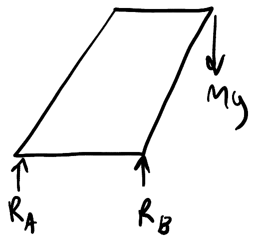
Method of Joints

1) Equilibrium analysis for whole object
   → Yields rxns at support
2) Identify special types of joints
3) Do the equilibrium analysis on all joints to identify axial forces on members



truss members can support 4 kN tension 1 kN compression.
Find largest load that can safely be supported.

NASA



$A_x = 0$ (we know this automatically)

$\Sigma F_y = 0$

$R_A + R_B = mg$

$\Sigma M_A = 0$

$R_B(1m) - mg(3) = 0$

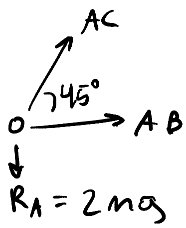
$R_B = 3mg$

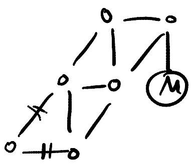
$R_A + 3mg = mg$

$R_A = -2mg$

No special Joints.

Joint A





$\Sigma F_x = 0$

$AB + AC\cos 45 = 0$

$AB = -AC\frac{\sqrt{2}}{2}$

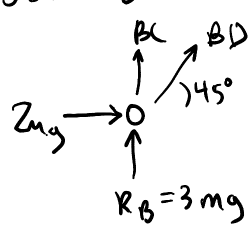
$\Sigma F_y = 0$

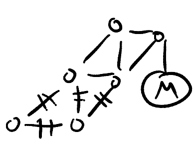
$AC\sin 45 - 2mg = 0$

$AC = 2\sqrt{2}\ mg$ tension

$AB = -2mg$ compression

Joint B





$\Sigma F_x = 0$

$2mg + BD\cos 45 = 0$

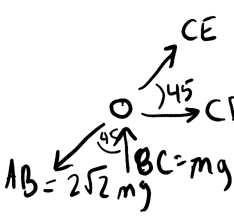
$BD = -2\sqrt{2}\ mg$ compression

$\Sigma F_y = 0$

$BC + 3mg + BD\sin 45 = 0$

$BC = -mg$ compression

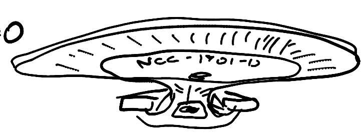
Joint C



$\Sigma F_x = 0$

$CD + CE\cos 45 - 2\sqrt{2}\,mg\sin 45 = 0$

$CD = mg$ (tension



$\Sigma F_y = 0$

$CE\sin 45 + mg - 2\sqrt{2}\overset{mg}{\cos 45} = 0$

$CE = \sqrt{2}\, mg$ (tension)

After remaining joint analysis

$DE = -mg$

$EF = mg$

$DF = -\sqrt{2}\, mg$

NUMB3RS

X-FILES

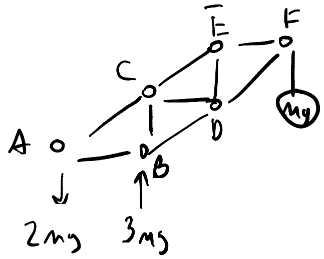
CSI:MIAMI

STAR TREK
TNG

er

Tension, Compression Trusses

Wednesday, February 07, 2007
8:01 AM



Tension	Compression
AC → $2\sqrt{2}$ mg	AB → 2mg
CB → $\sqrt{2}$ mg	BD → $2\sqrt{2}$ mg
CD → mg	BC → mg
EF → mg	DE → Mg
	DF → $\sqrt{2}$ mg

Largest value of m can be if each member can support max tension of 4 kN & comp. 1 kN.

$2\sqrt{2}$ mg is largest tension. $\leq 4000N$

$m \leq 144.16$ kg

$2\sqrt{2}$ mg is largest compression $\leq 1000N$

$m \leq 36.04$ kg

So max mass that truss can support is 36.04 kg.
(It's smaller)

Another method.

$$\begin{array}{c}
\begin{array}{cccccc} AC & AB & BC & & & EF \end{array} \\
\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & 0 & \cdots & & 0 \\ \frac{\sqrt{2}}{2} & 1 & 0 & \cdots & & 0 \\ & & & & & \\ & & & & & \end{bmatrix}
\end{array}
\begin{bmatrix} AC \\ AB \\ BC \\ BD \\ CD \\ CE \\ DE \\ DF \\ EF \end{bmatrix} = \begin{bmatrix} 2mg \\ 0 \\ \\ \\ \end{bmatrix}$$

$\Sigma F_x = 0$

$AB + \frac{\sqrt{2}}{2} AC = 0$

$\Sigma F_y = 0$

$\frac{\sqrt{2}}{2} AC = 2mg$

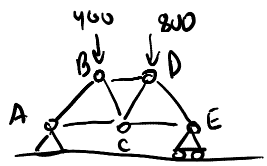
$$A \quad x = b$$

$$x = A^{-1} b$$

Method of Sections

1. Find rxns at supports. Equilibrium analysis of whole structure
2. Isolate sections from the truss and consider analysis just for those sections.

Warren Truss

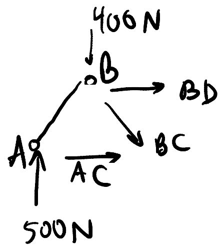


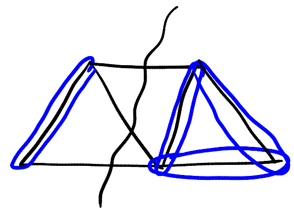
1. $A_y = 500\,N$

$\underbrace{E_y = 700\,N}$

From previous notes.

2. AC, BC, BD?





$$\Sigma F_x = 0 = AC + BD + BC\cos\alpha \quad 60°$$

$$\Sigma F_y = 0 = 500 - 400 - BC\sin 60$$

$115.47N = BC$ tension

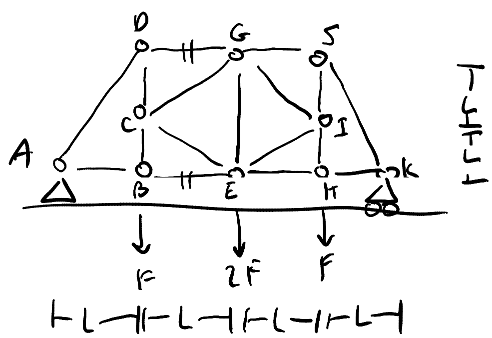
$$\Sigma M_B = 0 = -500(.5) + (1)AC\sin 60 = 0$$

$AC = 288.68\ N$ tension

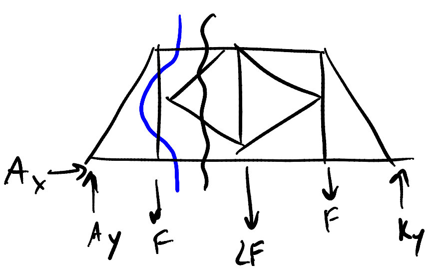
$$0 = 288.68 + BD + 115.47\cos 60$$

$-346.41N = BD$ Compression

Max # of things we can determine here is 3.



DG & BE?



This is a better cut than this ~.

$$\Sigma F_x = 0$$

$$A_x = 0$$

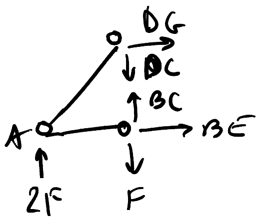
$$\Sigma F_y = 0$$

$$A_y - 4F + K_y = 0$$

$$\Sigma M_A = 0$$

$$-LF - 4LF - 3LF + K_y(4L) = 0$$

$$K_y = 2F$$



$$A_y - 4F + 2F = 0$$

$$A_y = 2F$$

$$\Sigma F_x = 0$$

$$DG + BE = 0$$

$$\Sigma M_D = 0$$

$$-LA_y + 2L\,BE = 0$$

$$2L\,BE = 2FL$$

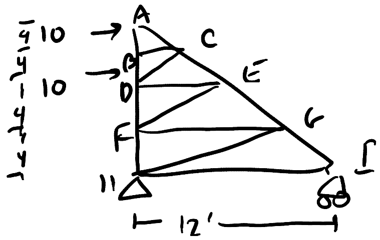
$$BE = F$$

$$DG = -F$$

Dynamic Systems (and the plot to XF: Beyond the Sea)

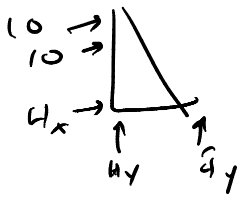
Friday, February 09, 2007

8:03 AM



HW #7 6.45, 6.50, 6.93, 6.106, 6.122(M) + 2 LED probs on blackboard.

Find EF + rxns



$\Sigma F_x = 0$

$H_x + 20 = 0$

$H_x = -20$

$\Sigma F_y = 0$

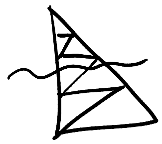
$H_y + I_y = 0$

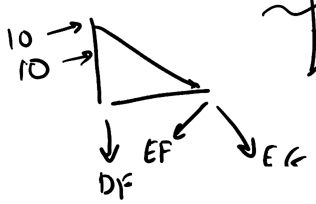
$\Sigma M_H = 0$

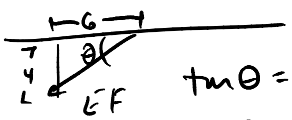
$-10(12) - 10(16) + I_y(12) = 0$

$I_y = 23.77$

$H_y = -23.77$

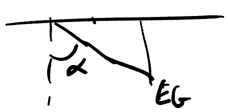






$\tan\theta = \frac{2}{3}$

$\theta = 33.69°$



$\tan\alpha = \frac{3}{4}$

$\alpha = 36.87°$

$\Sigma F_x = 0$

$20 - EF\cos\theta + EG\sin\alpha = 0$

$\Sigma F_y = 0$

$-DF - EF\sin\theta - EG\cos\alpha = 0$

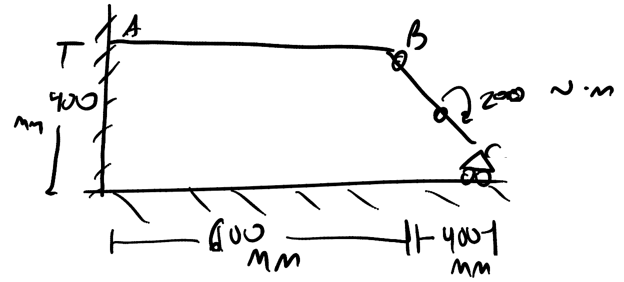
$\Sigma M_E = 0$

$-8 \cdot 10 - 4 \cdot 10 + 6\,DF = 0$

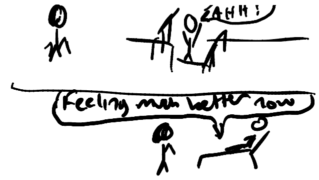
$DF = 20$

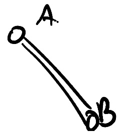
$EF = 4$

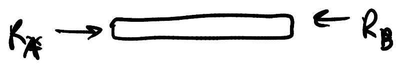
$EG = -27.8$



Not to scale.

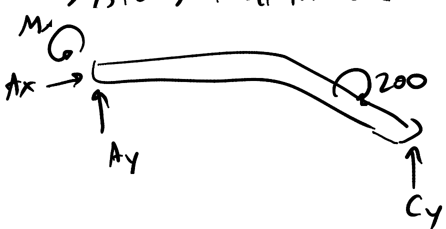






← 2 force system
Forces = in mag, opp in direction

Systems that are static (frames)
Systems that move (machines)



$\Sigma F_x = 0$

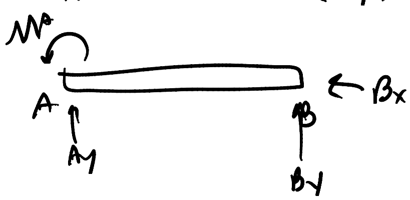
$A_x = 0$

$\Sigma F_y = 0$

① $A_y + C_y = 0$

$\Sigma M_A = 0$

② $M_A - 200 + 1_m(C_y) = 0$



$\Sigma F_x = 0$ $\quad$ $\Sigma F_y = 0$

$B_x = 0$ $\quad$ $A_y + B_y = 0$

$\Sigma M_A = 0$

$M_A + 0.6 \cdot B_y = 0$

$M_A = -.6 B_y$

$A_y = -B_y$

$B_y = C_y$

$-.6B_y - 200 + B_y = 0$

$.4B_y = 200$

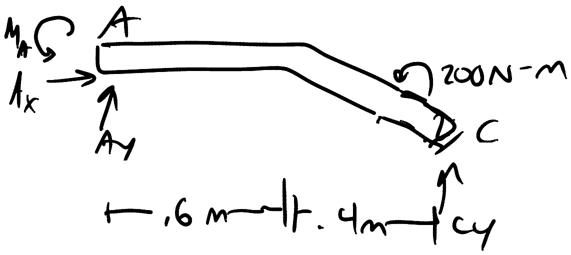
$B_y = 500\ N$
$A_y = -500\ N$

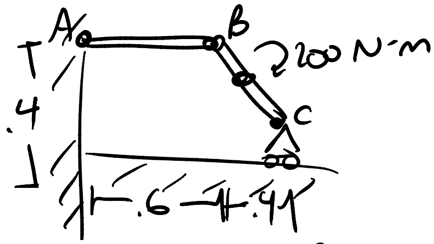
$C_y = 500N$
$M_A = -500N\cdot m$

Pulleys, Fx & Ma

Monday, February 12, 2007

8:03 AM





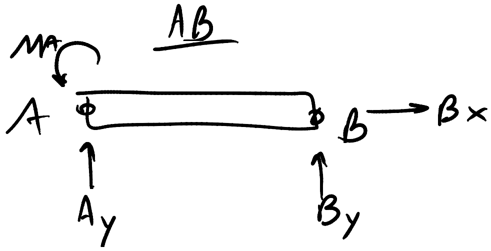
Find forces & couples & all that crap.

$\Sigma F_x = 0 \quad A_x = 0$

$\Sigma F_y = 0 \quad A_y + C_y = 0 \quad (1)$

$\Sigma M_A = 0 \quad M_A + C_y(1) - 200 = 0 \quad (2)$

<u>AB</u>



$\Sigma F_x = 0 \quad B_x = 0$

$\Sigma F_y = 0 \quad A_y + B_y = 0 \quad ; \quad B_y = -A_y$

$\Sigma M_A = 0 \quad M_A + B_y(.6) = 0 \quad (3)$

$M_A - .6A_y = 0$

$M_A = .6A_y$

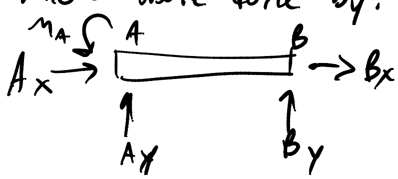
$200 = .6A_y + C_y$

$-C_y = A_y$

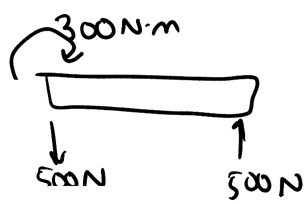
$200 = .6A_y - A_y$

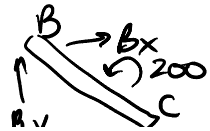
$$\boxed{\begin{array}{c} -500N = A_y \\ 500N = C_y \\ -300 N\cdot m = M_A \\ 500N = B_y \end{array}}$$

We know we're done by:

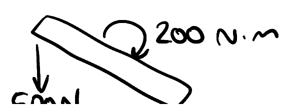


becomes

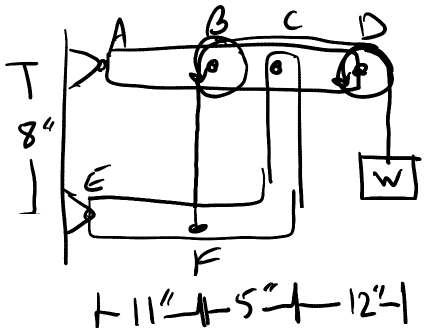




becomes



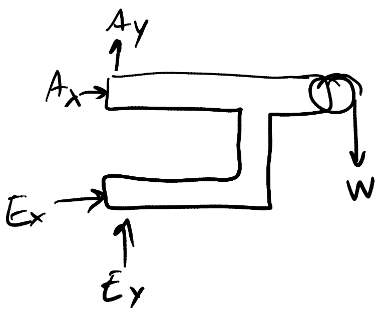
this is opposite b/c contact forces are = & opposite, & it has to cancel to make the original diagram, & they are internal forces (FBD's are external only)



W = 80 lb

r pulleys = 3"

AD forces?



$\Sigma F_x = 0 \quad A_x + E_x = 0$

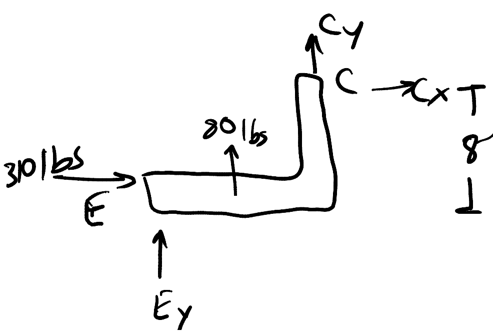
$\Sigma F_y = 0 \quad A_y + E_y = W$

$\Sigma M_A = 0 \quad E_x(8) - W(28+3) = 0$

$E_x(8) = 80 \cdot 31$

$E_x = 310 \text{ lbs}$

$A_x = -310 \text{ lbs}$



$\Sigma F_x = 0$

$C_x = -310 \text{ lbs}$

$\Sigma F_y = 0$

$E_y + 80 \text{ lbs} + C_y = 0$

$\Sigma M_E = 0$

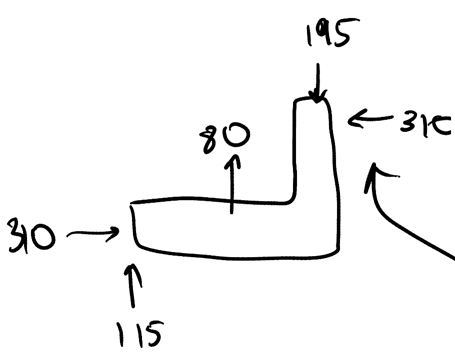
$80(11-3) + 16C_y - 8(C_x) = 0$

$C_y = -195 \text{ lbs}$

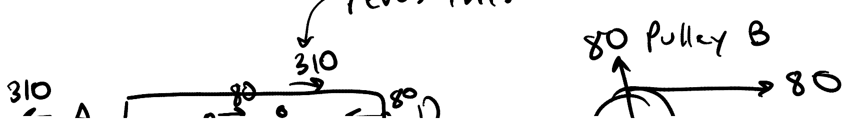
$E_y = 115 \text{ lbs}$

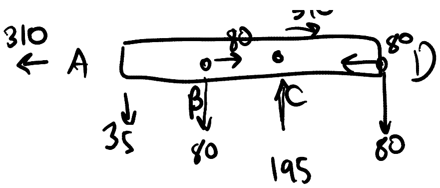
$A_y = 80 - 115 \text{ lbs}$

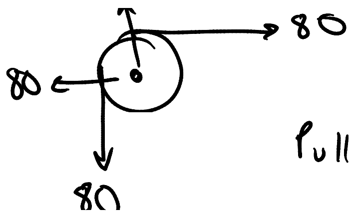
$A_y = -35 \text{ lbs}$



reverse this

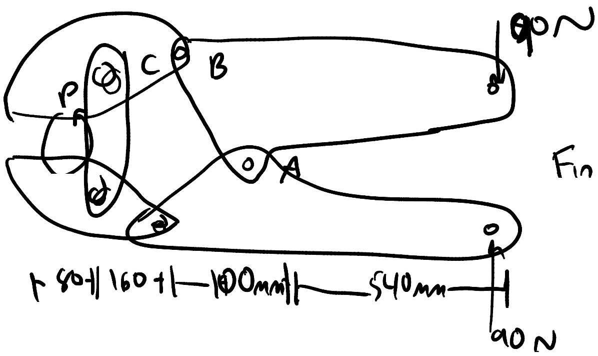


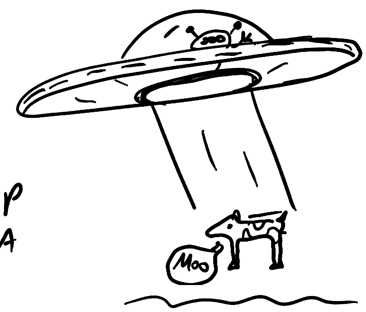




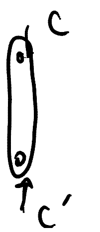
Pulley's in equilib.

This is AD ↻

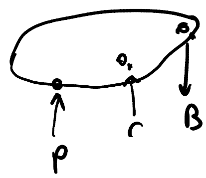




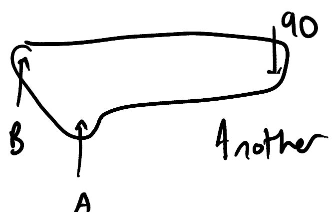
Find force @ P
A



Equal & opp.
rxns.



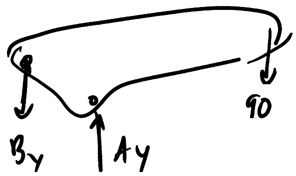
In 3 force members, the forces are ∥ or concurrent, which is why B is vertical. Downward b/c P & C are ↑.



Another 3 force system

Mechanics of Matls

Tuesday, February 13, 2007
8:06 AM

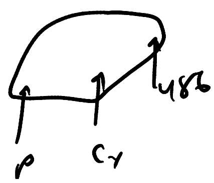


$\Sigma F_y = 0 = A_y - B_y - 90 = 0$

$\Sigma M_B = 0 \quad -.64 \cdot 90 + .1 A_y = 0$

$A_y = 576N$

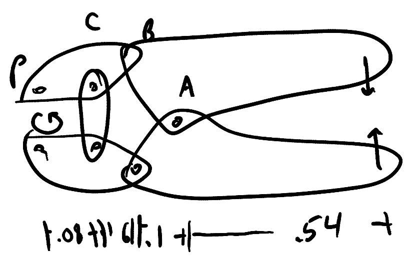
$B_y = 486\ N$

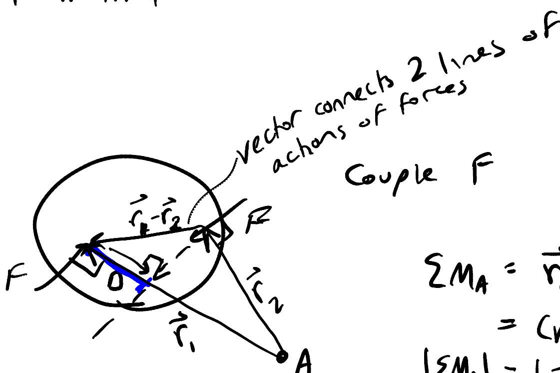


$\Sigma F_y = 0 \quad P + C_y + 486 = 0$

$\Sigma M_C = 0 \quad .16 \cdot 486 - .08 P = 0$

$P = 972\ N$





$\Sigma M_A = \vec{r}_1 \times F + \vec{r}_2 \times -F$

$= (\vec{r}_1 - \vec{r}_2) \times \vec{F}$

$|\Sigma M_A| = |\vec{r}_1 - \vec{r}_2| |\vec{F}| \sin\theta$

$|\Sigma M_A| = D |\vec{F}|$

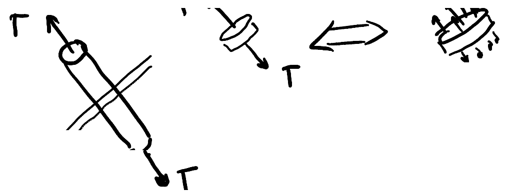
<u>Mechanics of Materials</u>

→ Objects

→ loads distributed on members of structures

→ Deformation

Stress – Strain



Stress - Average force per unit area.

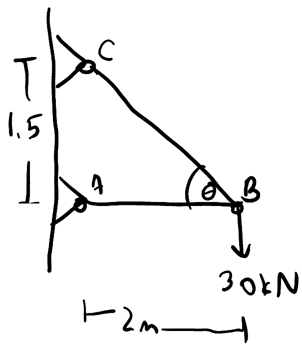
$$\sigma = \frac{T}{A} \quad \text{average } \frac{F\,/\,\text{Tension}}{\text{unit area } A}$$

Force is uniformly distribute over the cross section.

tensile (+)
compressive (−)

SI → $\frac{N}{m^2}$ ⇒ Pa
US → $\frac{lb}{in^2}$ ⇒

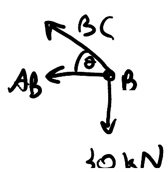
$MPa = 10^6\ Pa$
$k\frac{lb}{in^2} = ksi$



diameter BC = 2cm
maximum allowable stress for BC = 165 MPa

Plastic material deforms before it snaps.

$$\tan\theta = \frac{1.5}{2}$$
$$\theta = 36.87°$$



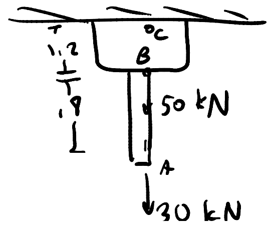
$\Sigma F_y = 0$
$BC \sin\theta = 30kN$
$BC = 50kN$
Member is in tension.

$$\sigma = \frac{T}{A} = \frac{50\ E3\ N}{\pi(.01)^2} = 159\ MPa$$, ≤ 165 MPa BC will hold it.

Safety Factor of a member, FS

$$FS = \frac{\sigma_y}{\sigma_{allowed}} = \frac{\text{yield stress}}{\text{allowed stress}}$$

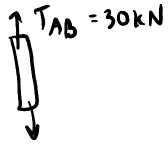
If you stretch it a little bit, it behaves like a spring.
If you pull hard enough, properties change.



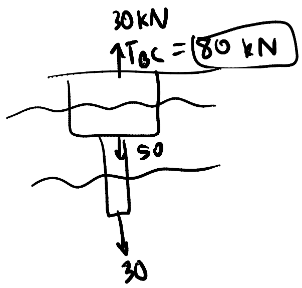
BC → 30 mm diameter
AB → 20 mm diameter

Up all night long

And there's something
very wrong
And I know it must
be late...
Been gone since yesterday
I'm not like you guys
I'm not like you



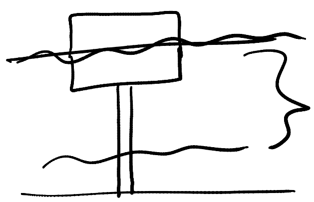
$\sigma_{AB} = \frac{T_{AB}}{A_{AB}} - 95.5 \pi R$

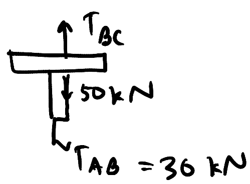


$\sigma_{BC} = 113 MPa$

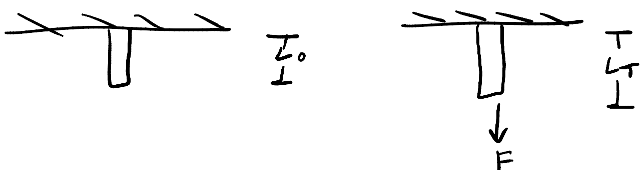
Darkness way
ordinary
Explanation
Information
Nice to know ya
Paranoia
Where's my mother
bio-father

Twelve Majestic Lies

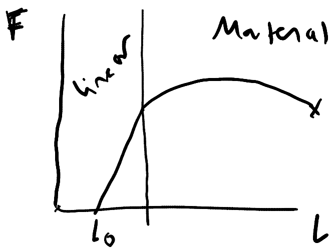




Strain used to classify materials according to properties
Using these new equations, we can determine solns to statically indeterminate systems.



$\delta = L_F - L_0$
Analagous to a spring



Material changes properties

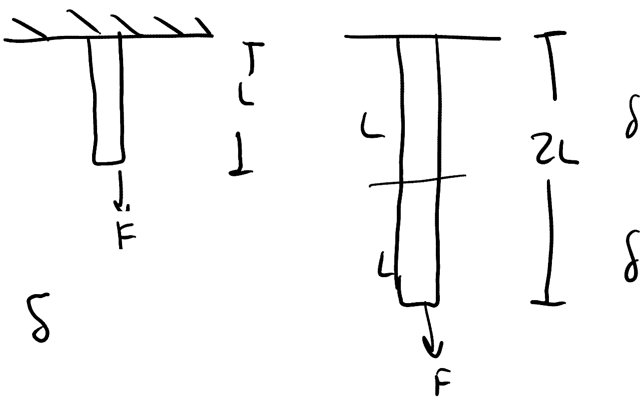
Load Deformation Diagram

FLYING SAUCER

⇒ Material
⇒ speed of loading
⇒ diameter of rods / dimensions
⇒ temperature

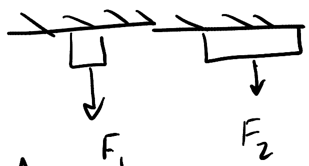
* assume forces along axis of rod.

<u>Dimensions</u>
Cross sectional area A
Length L



δ

Total deformation = 2δ

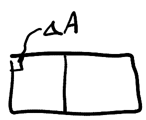


L, A_1 $\quad$ L, A_2

$A_2 = 2A_1$

δ, Find F_2 s.t. rod 2 stretches the same amt delta δ.

$\Sigma \frac{F}{\Delta A} = \frac{F}{A_1/\Delta A}$



$\frac{F}{\Sigma \Delta A} = \frac{F}{2A_1/\Delta A}$

A_1 $\qquad$ A_2

F must be larger on 2nd rod b/c it has do = first rod. So must be 2x as large.

$$\frac{F}{A_1/\Delta A} = \frac{\cancel{2}F}{\cancel{2}A_1/\Delta A} \quad \checkmark$$

$\sigma = \frac{F}{A}$ $\quad$ Normalize deformation we observe w.r.t. strain.

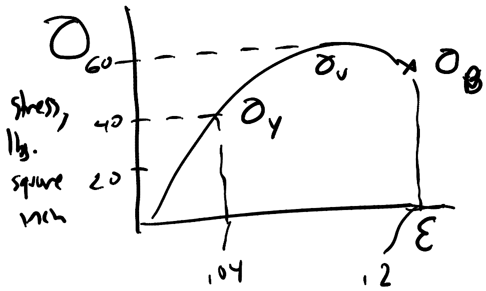
$$\varepsilon = \frac{\delta}{L} \quad \frac{\text{deformation}}{\text{original length}} = \frac{m}{m} = \text{constant}$$

Strain has no units.

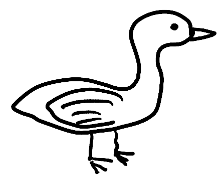
Stress/strain diagram

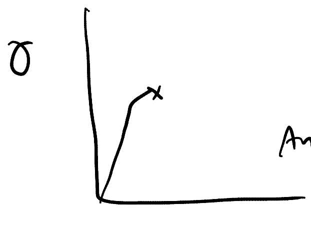
Property of the material, not of the length or dimensions.

Aluminum



Ductile - things that stretch

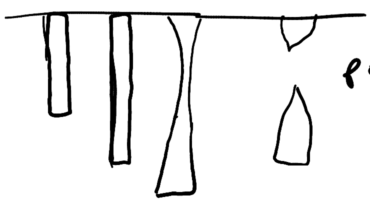




Another material.
Brittle - things that don't.

Material breaks very shortly after linear line ends.

σ_y yield stress / strength
σ_u ultimate stress (max stress)
σ_B breaking stress (where it breaks)



point where material breaks is really narrow.
Cross sectional area increases as material narrows. To provide same force to break, need to add force b/c A is bigger.

$$\frac{F_T}{A_B}$$

Brittle materials don't narrow

Ductility

→ % elongation $= 100 \dfrac{L_B - L_0}{L_0}$

→ % reduction in area $= 100 \dfrac{A_0 - A_B}{A_0}$

For engineering applications, we usually design members for small deformations.

Hook's Law: $\sigma = E\varepsilon$, E = Young's modulus of a material.
$= Pa$

Deformations are reversible

Elastic materials

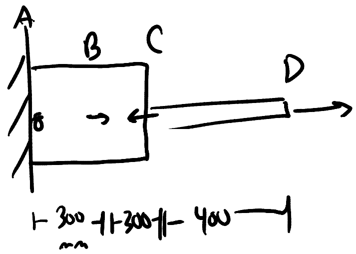
Plastic materials do not return to original length.

Hooke's Law

Friday, February 16, 2007
8:04 AM

HW #8 13.29, 13.40, 13.53, 13.57

Hook's Law $\sigma = E\varepsilon$ E = Young's Modulus, property of material



$A_{ABC} = 600\ mm^2$

$A_{CD} = 200mm^2$

$F_B = 500kN$

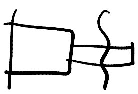
$F_C = 300\ kN$

$F_D = 200\ kN$

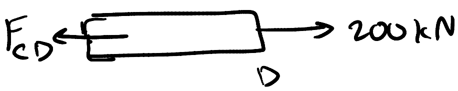
$E = 200GPa$

$\sigma = E\varepsilon$

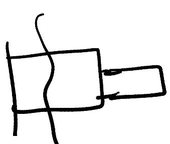
$\frac{F}{A} = E\frac{\delta}{L}$ $\delta = \frac{FL}{AE}$ $\delta_T = \delta_{AB} + \delta_{BC} + \delta_{CD}$

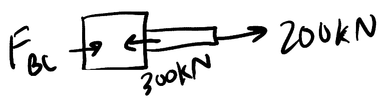


$$\delta_T = \frac{F_{AB}L_{AB}}{A_{AB}E} + \frac{F_{BC}L_{BC}}{A_{BC}E} + \frac{F_{CD}L_{CD}}{A_{CD}E}$$



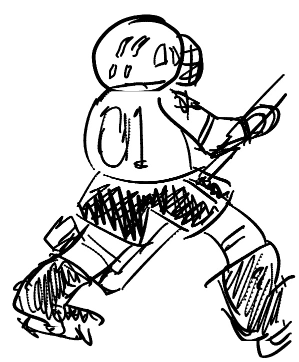
$F_{CD} = 200kN$ to the left

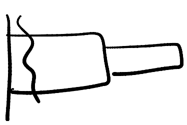


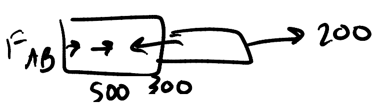


$F_{BC} = 300 - 200\ kN$

$F_{BC} = 100\ kN$ so points to right







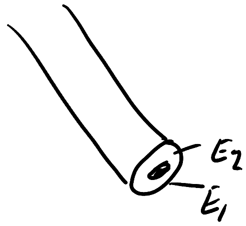
$F_{AB} + 500 - 300 + 200 = 0$

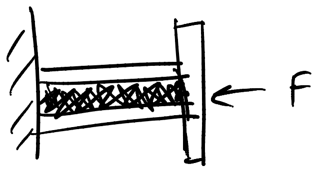
$F_{AB} = -400\ kN$ so points to left

compression

$$\delta_T = \frac{(400)(300)}{(300)(200)} - \frac{(100)(300)}{(300)(200)} + \frac{(200)(400)}{(200)(200)}$$

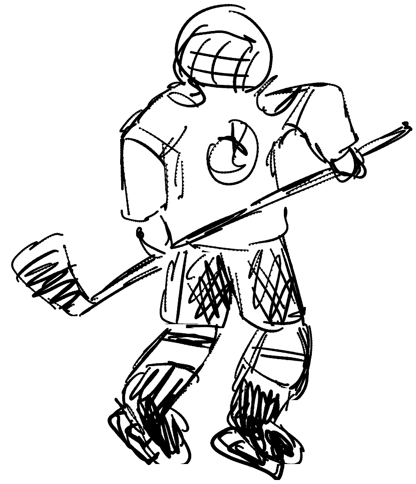
$\delta_T = 2.75\ mm$





What part of F is being supported by each material

$$F = F_1 + F_2$$



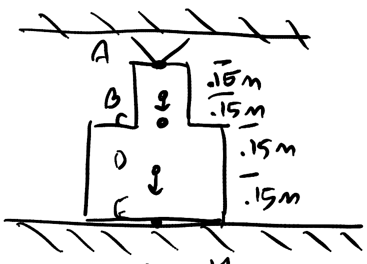
$$\delta_1 = \delta_2 = \delta$$

$$\delta_1 = \frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} = \delta_2$$

$$F_1 + F_2 = F$$

$$F_2 = \frac{F A_2 E_2}{A_1 E_1 + A_2 E_2} \; ; \; F_1 = \frac{F A_1 E_1}{A_1 E_1 + A_2 E_2}$$

They need to sum to 1

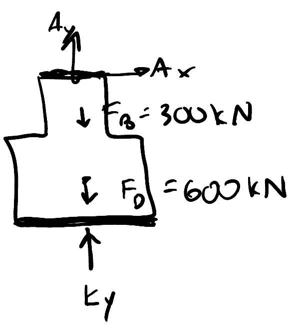


$A_{AC} = 250 \text{ mm}^2$ $\quad E = 200 \text{ GPa}$

$A_{CE} = 400 \text{ mm}^2$

$F_B = 300 \text{ kN}$

$F_D = 600 \text{ kN}$



$$\Sigma F_x = A_x = 0$$

$$\Sigma F_y = A_y - 300 - 600 + E_y = 0$$

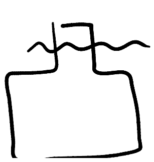
$$A_y + E_y = 900$$

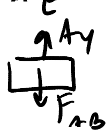
$$\Sigma M_A = 0$$

$\delta_T = 0$ b/c fixed top & bottom

$$\delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 0$$

$$\frac{FL}{AE} = \delta$$



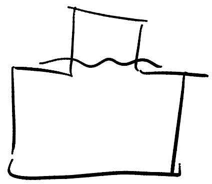


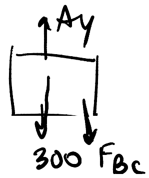
$$A_y = -F_{AB}$$

$$E_y - 900 = F_{AB}$$

$\delta \quad (F_{AB}) L_{AB}$

$$\delta_{AB} = \frac{(F_{AB})\, L_{AB}}{A_{AB}\, E}$$





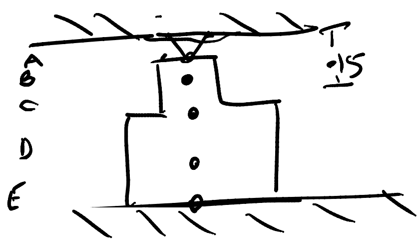
$$F_{BC} = A_y - 300$$

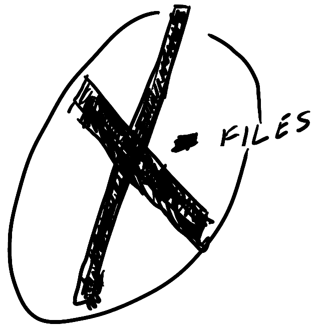
$$\delta_{BC} = \frac{(-F_{AB} - 300)\, L_{BC}}{A_{BC}\, E}$$

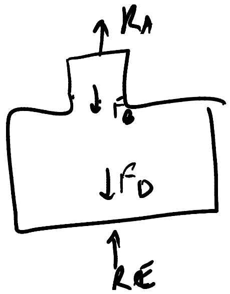
$A_y = 323.08$ kN
$E_y = 576.92$ kN

Deformation, Kinematics

Monday, February 19, 2007
8:03 AM







$\Sigma F_y = 0$

$A_{AC} = 250\ mm^2$

$A_{CE} = 400\ mm^2$

$F_B = 300\ kN$

$F_D = 600\ kN$

$E = 200\ GPa$

$R_A + R_E = 900\ kN$

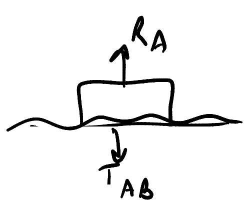
$\delta_T = 0 = \delta_1 = \delta_2$

$\sigma = E\varepsilon$

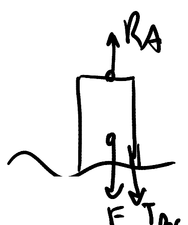
$\frac{F}{A} = E\frac{\delta}{L}$

$\frac{FL}{AE} = \delta$

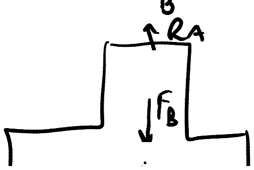
$\delta_T = 0 = \frac{T_{AB} L}{A_{AB} E} + \frac{T_{BC} L}{A_{BC} E} + \frac{T_{CD} L}{A_{CD} E} + \frac{T_{DE} L}{A_{DE} E}$



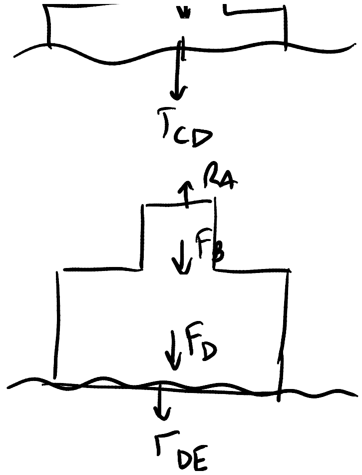
$\Sigma F_y : T_{AB} = R_A$



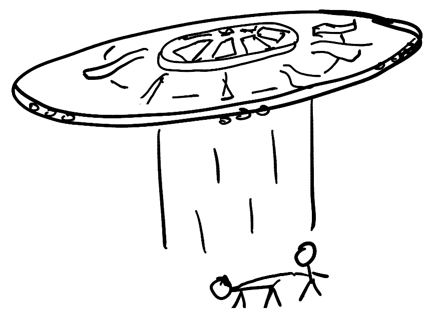
$T_{BC} = R_A - 300$



$T_{CD} = R_A - 300$



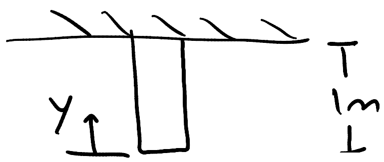
$T_{DE} = R_A - 900$



Solve for R_A with this.

$R_A = 323.08$ kN , then $R_E = 576.92$ kN

This ends required material for midterm & design project.



steel $\rho = 7.85E3$ kg/m^3

$E = 200$ GPa

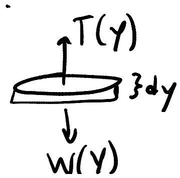
Find deformation induced by weight of bar.

Weights are a body force, distributed throughout load.

Eventually, it's all right to ignore weight of the structure.
Deformation is small due to weight.

$$\sigma(y) = \frac{w(y)}{A} = \rho y g = \gamma y$$

weight density $= \rho g$



$$w(y) = \underbrace{\rho}_{\text{density}} \underbrace{A \cdot y}_{V} \vec{g}$$

$$\frac{\partial \delta}{\partial y} = \frac{\sigma(y)}{E}$$

$$\boxed{\partial \delta = \frac{\sigma(y)}{E} \partial y}$$

$$\int \partial \delta = \int \frac{\sigma(y)}{E} \partial y$$

$$\delta = \int_{y=0}^{y=L} \frac{\sigma(y)}{E} \partial y = \frac{\gamma}{E} \left. \frac{y^2}{2} \right]_0^L = \frac{\gamma}{E} \frac{L^2}{2} = 1.12E{-7}\,m$$

Yay, MATH!

NUMB3RS

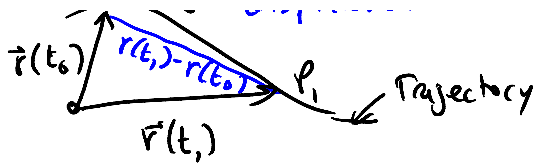
EITHER/OR

IF/THEN

BEFORE/AFTER

Kinematics

displacement



$\vec{r}(t)$ is defined as well as twice differentiable in $[t_0, t_1]$

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$$
$$\vec{v} = \frac{\partial \vec{r}(t)}{\partial t} = \frac{\partial x}{\partial t}\hat{\imath} + \frac{\partial y}{\partial t}\hat{\jmath} + \frac{\partial z}{\partial t}\hat{k}$$

$$\vec{a}(t) = \frac{\partial \vec{v}}{\partial t}$$

$$\vec{w} = s(t)\,\vec{v}(t)$$
$$\frac{\partial \vec{w}}{\partial t} = s'(t)\vec{v}(t) + s(t)\frac{\partial \vec{v}(t)}{\partial t}$$
$$s'(t)\left\langle \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}\right\rangle + s(t)\langle x''(t), y''(t), z''(t)\rangle$$

He says it's the chain rule but it's really just how you take the derivative of a multiple.

Motion along a straight line

$\hat{e}$ = unit vector in direction of a line

$$\vec{r}(t) = s(t)\,\hat{e}$$
$$\vec{v}(t) = V(t)\,\hat{e}$$
$$\vec{a}(t) = A(t)\,\hat{e}$$

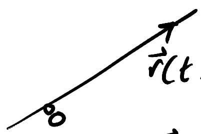
Velocity, Force

Tuesday, February 20, 2007
8:02 AM

$\vec{r}(t)$

$\vec{v}(t) = \frac{\partial}{\partial t}\vec{r}(t)$

$a(t) = \frac{\partial}{\partial t}\vec{v}(t)$



$\vec{r}(t) = s(t)\hat{e}$

$\vec{v}(t) = \frac{\partial}{\partial t}(s(t)\hat{e}) = \frac{\partial s(t)}{\partial t}\hat{e} + 0$ b/c $\hat{e}$ is constant w.r.t. time

$= v(t)\hat{e}$

$\vec{a}(t) = \frac{\partial}{\partial t}(v(t)\hat{e}) = \frac{\partial v(t)}{\partial t}\hat{e}$

$= a(t)\hat{e}$

ex

$s(t) = 6 + \frac{1}{3}t^3$

$v(t) = t^2$

$a(t) = 2t$

$\Sigma\vec{F}(t) = m\vec{a}(t)$

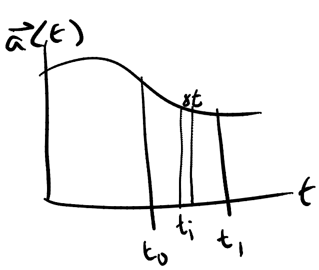
$a(t)$

$v(t) = \int a(t)\,dt$

$v(t) = \_\_\_\_ + C_1$ ← have to be given additional info to specify this uniquely.

$s(t) = \int v(t)\,dt$

$s(t) = \_\_\_\_ + C_2$



$\vec{a}(t) = \frac{d\vec{v}}{dt}$

$\int_{t_0}^{t_1}\vec{a}(t)\,dt = \int_{v_0}^{v_f} d\vec{v}$

$\int_{t_0}^{t_1}\vec{a}(t)\,dt = V_f - V_0$

$\vec{v}(t) - \vec{v}_0 = V_f - V_0$

N subintervals of equal length

Area under acceleration curve is velocity.

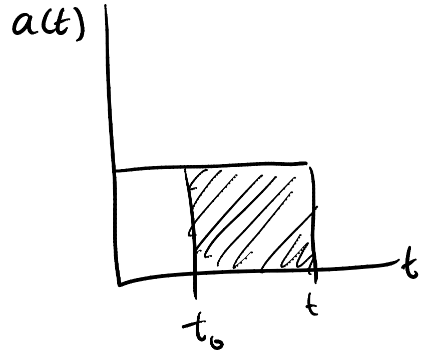
$\Delta v = \vec{a}(t_i)(t_{i+1} - t_i)$

$\sum_{i=1}^{N}\Delta v_i$

$\lim \Sigma = \int^{t_1}$

$$\lim_{N \to \infty} \sum = \int_{t_0}^{t_1}$$

$$s(t) - s_0 = \int \vec{v}(t)\,dt$$



$a(t) = a \quad ; [t_0, t]$

$$v(t) - v_0 = \int_{t_0}^{t} a\,dt$$

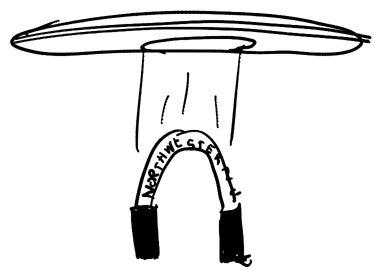
$$\boxed{v(t) = v_0 + a(t - t_0)}$$

$$s(t) - s_0 = \int_{t_0}^{t} v(t)\,dt$$

$$s(t) - s_0 = \int_{t_0}^{t} [v_0 + a(t - t_0)]\,dt$$

$$s(t) - s_0 = v_0 t\Big]_{t_0}^{t} + \frac{1}{2}a(t-t_0)^2\Big]_{t_0}^{t}$$

$$\boxed{s(t) = s_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2}$$



$$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$$

$$\int_{s_0}^{s} a\,ds = \int_{v_0}^{v} v\,dv$$

$$as\Big]_{s_0}^{s} = \frac{1}{2}v^2\Big]_{v_0}^{v_s}$$

$$\boxed{a(s - s_0) = \frac{1}{2}v_s^2 - \frac{1}{2}v_0^2}$$

ex

$v(t) = 200 - 2t^2$ m/s

At $t = 3$s, $s(t) = 600$ m. Find $s(6)$ & $a(6)$

$$a(t) = \frac{dv(t)}{dt}$$

$$a(6) = -4t = -4(6) = -24 \text{ m/s}^2$$

$$\frac{ds}{dt} = v(t)$$

$$\int ds = \int v(t)\,dt$$

$$s\Big]_{s_0}^{s} = 200t - \frac{2}{3}t^3\Big]_{t_0}^{t}$$

$$s - s_0 = 200(t - t_0) - \frac{2}{3}(t - t_0)^3$$

$$s(6) = 200(6-3) - \frac{2}{3}(6-3)^3$$

$$s(6) - 600 = 200(6-3) - \tfrac{2}{3}(6-3)^3$$
$$s(6) = \text{stuff}$$
$$s(6) = 1074$$

ex

$t = 0$, $s_0 = 6m$, $v_0 = 2 m/s$. From $t=0$, $t=6$, $a = 2 + 2t^2$, m/s^2

From $t=6$, $t = \underset{\text{object comes to rest}}{?}$, $a = -4 \, m/s^2$

Find total time of travel, total distance traveled.

$$v_f - v_0 = a(t - t_0)$$
$$v(6) - v(0) = \int_{t=0}^{t=6} a(t)\,dt$$
$$v(6) = 2 + \int_{t=0}^{t=6} 2 + 2t^2 \, dt$$
$$v(6) = 2 + 2t + \tfrac{2}{3}t^3 \Big]_0^6$$
$$v(6) = 158 \, m/s$$
$$-158 = -4(t - 6)$$
$$t = 45.5 \, s$$

Method of Sections

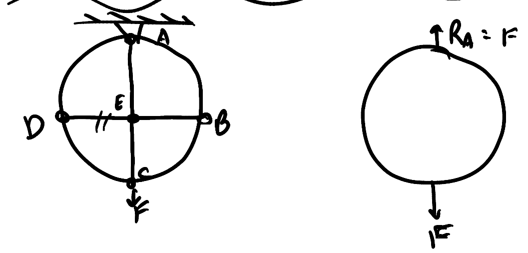
Wednesday, February 21, 2007
8:04 AM

EXAM

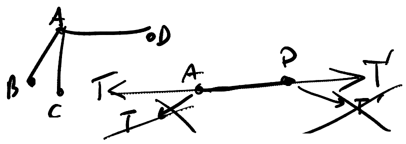
1st Problem is truss — Method of joints, method of sections
Find axial forces

2nd Problem is frame — Look at class examples & HW probs.
1st step, consider object as whole & obtain forces
2nd step, separate members & do equilib analysis

3rd Problem is deformation — Thinking problem. No autopilot. Not a straight application of Hook's Law



Method of sections

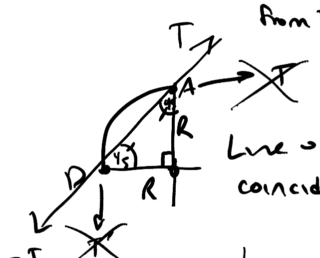


The forces are = & opp in direction.

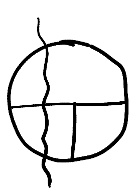
Loading occurs on those 2 forces, = & opp. Line of action of force coincides w/ member itself

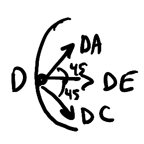
We know it is on line of action through AD (Skinner)

A o→ AD (Skinner) coincides w/ direction of member. we know direction from location of member AD. (Skinner)



Line of action no longer coincides w/ member.





$\Sigma F_x = 0 = DE + DA\cos 45 + DC\cos 45$

$\Sigma F_y = 0 = DA\sin 45 - DC\sin 45$

$DC = DA$

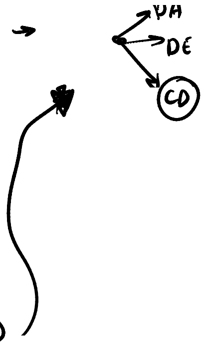
$DE = -2\,DA\cos 45$

IF the forces are concurrent, & they all intersect @ 1 pt, you can get 2 linearly independent eqns.

Joint E

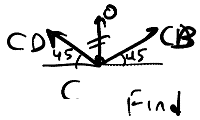
CD →

$\downarrow$ CD $\rightarrow$



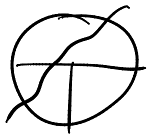
$EC = 0$

$\Sigma F_y = 0$
$EC = 0$

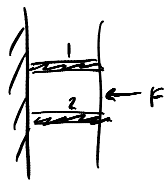


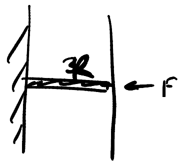
Find CD

or

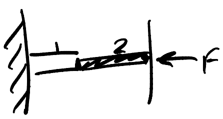


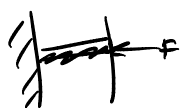
And you can find the forces.





(*) $F_1 + F_2 = F_R$
(**) $\delta_1 = \delta_2 = \delta_R$





(**) $F_1 = F_2 = F_R$
(*) $\delta_1 + \delta_2 = \delta_R$

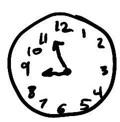
$E_R(E_1, E_2)$

$E_R = E_1 + E_2$ for one of the 2 sys's
$\frac{1}{E_R} = \frac{1}{E_1} + \frac{1}{E_2}$ for the other. } something like that

---

$\int_{v_0}^{v_f} v\,dv = \int_{s_0}^{s} a\,ds$ If a is a function of position, this still works

$$F = \frac{G m_1 m_2}{s^2}$$



$$W = mg = \frac{G m_E M}{R_E^2}$$

$$\frac{F}{W} = \frac{R_E^2}{s^2}$$

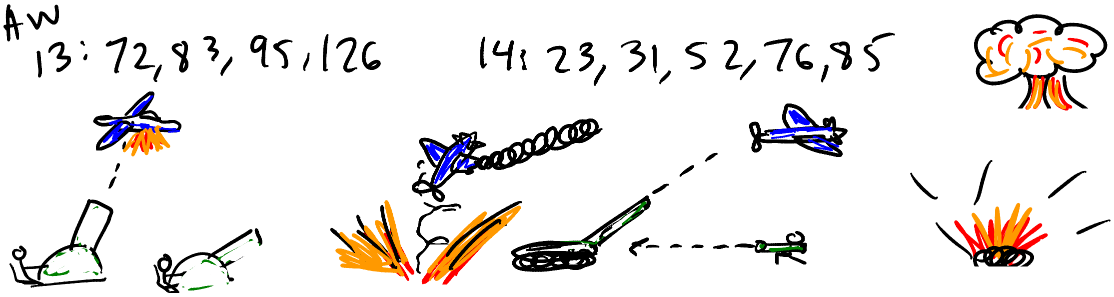
$$ma = F = \frac{R_E}{s^2}(W = mg)$$

$$a(s) = \frac{R_E^2}{s^2} g$$

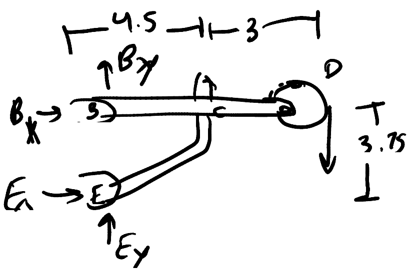
Example Prob, Exam Review

Friday, February 23, 2007
8:02 AM

AW
13: 72, 83, 95, 126   14: 23, 31, 52, 76, 85



The Above Scene Depicts The Midterm



$\Sigma F_x = 0 = B_x + E_x$

$B_x = -E_x$

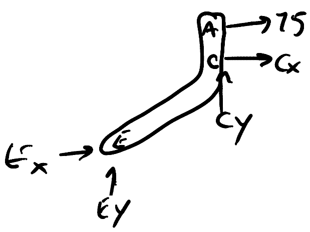
$\Sigma F_y = 0 = E_y + B_y - 75$

$75 = E_y + B_y$

$\Sigma M_B = 0 = -75(7.5 + 1.25) + 3.75(E_x)$

$175 \text{ lb} = E_x$

$B_x = -175 \text{ lb}$



$\Sigma F_x = 0 = C_x + E_x + 75$

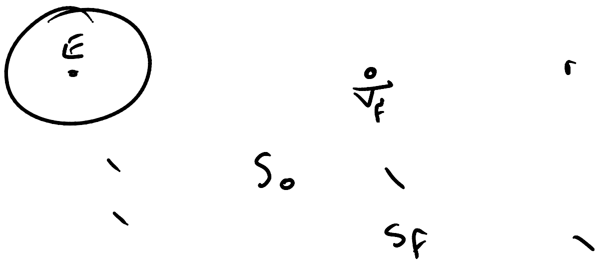
$\Sigma F_y = 0 = C_y + E_y$

$75 - E_y = B_y$

Projectile Motion

Monday, February 26, 2007
8:01 AM

$$a(s) = \frac{R_E^2}{s^2} g$$



$$\int_{v_0}^{v_f} v\,\partial v = \int_{s_0}^{s_f} a(s)\,\partial s$$

$$\frac{1}{2}v^2\Big]_{v_0}^{v_f} = \int_{s_0}^{s_f} a(s)\,\partial s$$

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = \int_{s_0}^{s_f} \frac{R_E^2}{s^2} g\,\partial s$$

$$-\frac{1}{2}v_0^2 = \left[-\frac{R_E^2}{s} g\right]_{s_0}^{s_f}$$

$$\frac{1}{2}v_0^2 = \frac{gR_E^2}{s_0} - \frac{gR_E^2}{s_f}$$

$$v_0 = \sqrt{2gR_E^2\left(\frac{1}{s_0} - \frac{1}{s_f}\right)}$$

This is how you get off of Earth $\lim_{s_f \to \infty} v_0 = \sqrt{\frac{2gR_E^2}{s_0}}$

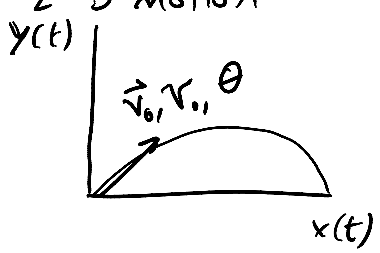
$\vec{r}(t)$
$\vec{v}(t)$
$\vec{a}(t)$

Linear motion expressed in terms of $\hat{e}$. Now we express them in terms of Cartesian components b/c of multidimensional motion.

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$
$$\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k}$$
$$\vec{a}(t) = a_x(t)\hat{i} + a_y(t)\hat{j} + a_z(t)\hat{k}$$

Projectile ~~Vomiting~~ Motion

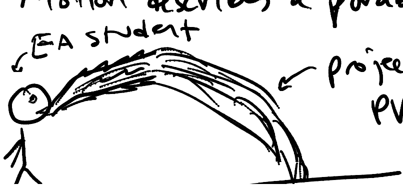
2-D motion



$v_0 = |\vec{v}_0|$

Objects only subjected to acceleration due to gravity (in y direction)

Motion describes a parabola. Acceleration given by $-g$ (in y)
0 (in x)



projectile vomit
$PV = -\frac{1}{2}(x+4)^2$

x direction

$\vec{a}_x(t)=0$

$V_x(t)$?

$\vec{a}_x(t)=0$

$$\int_{V_{0x}}^{V_x(t)} \partial v = \int_{t=0}^{t_f} a_x(t)\, dt$$

$$V_x(t) - V_{0x} = 0$$

$$V_x(t) = V_{0x} = V_0 \cos\theta$$

$$\int_{x_0}^{x_t} \partial x = \int_{t=0}^{t_f} V_x(t) dt$$

$$x(t) - x_0 = V_0 \cos\theta\, t$$

y direction

$\vec{a}_y(t) = -g$

$V_y(t)$?

$a_y(t) = -g$

$$\int_{V_{0y}}^{V_y(t)} \partial v = \int_{t=0}^{t_f} a_y(t)\, dt$$

$$V_0(t) = V_{0y} - gt$$

$$\int_{y_0}^{y_t} \partial y = \int_{t=0}^{t_f} V_y(t) dt$$

$$y(t) - y_0 = \int_{t=0}^{t=t_f} V_{0y} - g t\, dt = V_{0y} t - \frac{1}{2} g t^2 \Big]_{t=0}^{t=t_f}$$

$$y(t) = y_0 + V_{0y}(t) - \frac{1}{2} g t^2$$

Baseball pitcher releases fastball at 90 mph. (MR DRAGONETTI!)
Some ∡, ∡ of release above horizontal.

If pitcher throws strike @ ∡ 1°, vs 2°. (So pitchers are really skilled, predicting a few inches. Baseball is a game of inches)

$$\frac{90 \text{ miles}}{\text{hour}} \left(\frac{1 \text{ hour}}{3600 \text{ sec}}\right)\left(\frac{5400 \text{ ft}}{1 \text{ miles}}\right) = 135 \text{ ft/sec}$$

$x(t) = V_0 \cos\theta\, t$

$$\boxed{t = \frac{x(t)}{V_0 \cos\theta}} = \frac{0.44}{\cos\theta}$$

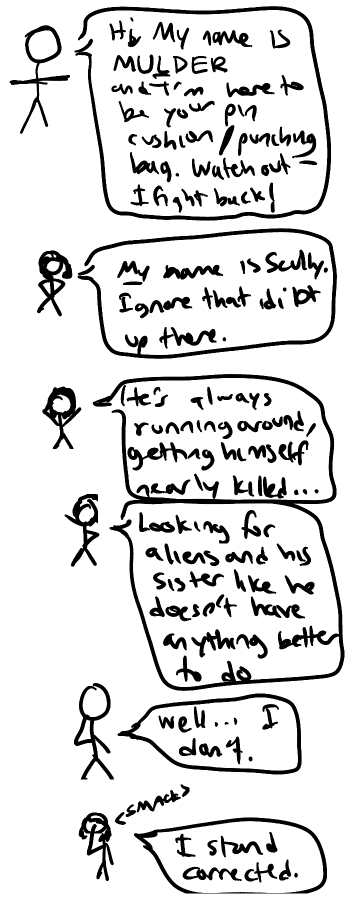
$$\boxed{y(t) = y_0 + v_0 \sin(\theta)\, t - \tfrac{1}{2} g t^2}$$

$$y(t) = y_0 + (135)(.44)\tan\theta - \tfrac{1}{2} g \left(\frac{.44}{\cos\theta}\right)^2$$

$$y(t)\Big|_{\theta=1} = 3.9' \quad \text{In strike zone.}$$

$$y(t)\Big|_{\theta=2} = 4.9' \quad \text{Not in strike zone}$$

Projectile motion w/ v_0 given. What ∡ above horizontal maximizes range of projectile?



$$x(t) = v_0 (\cos\theta) t$$

$$t(\theta) = \ ?$$

$$y(t) - y_0 = v_0 \sin\theta\, t - \tfrac{1}{2} g t^2$$

$$0 = v_0 \sin t - \tfrac{1}{2} g t^2$$

$$0 = t(v_0 \sin\theta - \tfrac{1}{2} g t)$$

$$t = 0, \quad \boxed{t = \frac{2 v_0 \sin\theta}{g}}$$

$$x(t) = (v_0 \cos\theta)\left(\frac{2 v_0 \sin\theta}{g}\right)$$

$$\cancel{x}(t) = \frac{2 v_0^2}{g} \sin\theta\cos\theta$$

$$\frac{\partial x}{\partial\theta} = 0$$

$$0 = \frac{2 v_0^2}{g}(\cos^2\theta - \sin^2\theta)$$

$$\sin^2\theta = \cos^2\theta$$

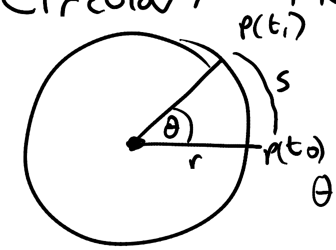
$$\sin\theta = \cos\theta$$

$$\theta = 45^\circ$$

Circular Motion

Tuesday, February 27, 2007
8:04 AM

Circular MOTION



f is fixed. position can be described as an ∡ θ.

$\theta(t), \quad t \in [t_0, t_1]$

angular velocity

$\omega(t) \equiv \frac{\partial \theta(t)}{\partial t}$

angular acceleration

$\alpha(t) = \frac{\partial \omega(t)}{\partial t}$

$t=0, \ \omega(0)=0$

$t \in [0,3], \ \alpha(t) = 6t$ rad/s²

$t > 3, \ \alpha(t) = -3$ rad/s² until $\omega_f = 0$

a) max angular velocity?

We know it will happen at $t = 3s$

$\int_{\omega(0)}^{\omega(3)} \partial\omega = \int_{t=0}^{t=3} 6t \, \partial t \ ; \ \omega(3) - \cancel{\omega(0)}^{0} = [3t^2]_0^3$

$\omega(3) = 27$ rad/s

b) Total ∡ traveled?

$t \in [0,3], \ \omega(t) = 3t^2$

$t > 3, \quad \int_{\omega(3)}^{\omega(t)} \partial\omega = \int_{t=3}^{t} \alpha(t) \, \partial t$

$\omega(t) = \omega(3) + \left(-3t \right]_{t=3}^{t=t})$

$\omega(t) = 27 + (-3t + 9)$

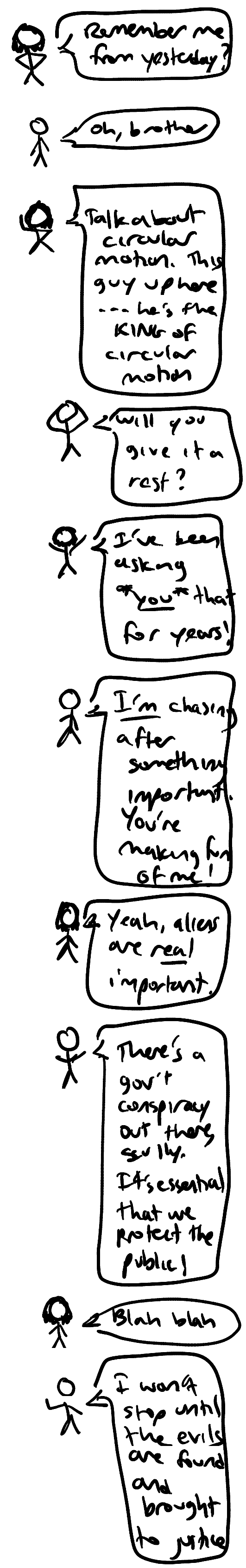
max found at $\rightarrow \omega(t) = -3t + 36$

$\int_{\theta(0)}^{\theta(12)} \partial\theta = \int_{t=0}^{t=12} \omega(t) \, \partial t$

$\theta(12) = \int_{t=0}^{t=3} 3t^2 \, \partial t \ \int_{t=3}^{t=12} (36 - 3t) \, \partial t$

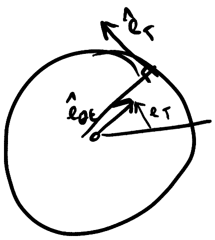
$\theta(12) = t^3 \Big]_0^3 + 36t - \frac{3t^2}{2} \Big]_3^{12}$

$\theta(12) = 148.5$ rad



$s = \theta r$ with this, can find distance traveled.

$$\vec{r}(t) = r(\cos(\theta)t\hat{\imath} + \sin(\theta)(t)\hat{\jmath})$$
$$= r \cdot \hat{e}_{\theta(t)}$$
$$\vec{v}(t) = \frac{\partial \vec{r}(t)}{\partial t} = r\frac{\partial \hat{e}_{\theta(t)}}{\partial t} = r\left(-(\sin\theta)t\hat{\imath} + (\cos\theta)t\hat{\jmath}\right)\frac{\partial\theta(t)}{\partial t}$$
$$\vec{v}(t) = r\omega(t)\left(-(\sin\theta)t\hat{\imath} + (\cos\theta)t\hat{\jmath}\right)$$
$$\vec{v}(t) = v(t)\,\hat{e}_T$$
$$v(t) = r\omega(t)$$
$$\hat{e}_T = -(\sin\theta)t\hat{\imath} + (\cos\theta)t\hat{\jmath}$$

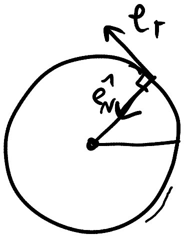


$\hat{e}_{\theta t}$ & $\hat{e}_T$ are $\perp$

tangent to Θ in direction of motion.

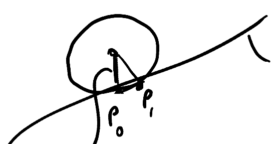
Velocity is always in direction of motion.

$$\vec{a}(t) = \frac{\partial \vec{v}(t)}{\partial t} = \frac{\partial v(t)}{\partial t}\hat{e}_T + \frac{\partial \hat{e}_T}{\partial t}v(t)$$
$$= a(t)\hat{e}_T + v(t)\left(-(\cos\theta)t\hat{\imath} - (\sin\theta)t\hat{\jmath}\right)\frac{\partial\theta(t)}{\partial t}$$
$$= a(t)\hat{e}_T + v(t)\omega(t)(\hat{e}_N)$$



Circular motion

$$s = r\theta$$
$$v = r\frac{\partial\theta}{\partial t} = r\omega$$
$$a_T = r\frac{\partial\omega}{\partial t} = r\alpha$$
$$a_N = v\omega$$
$$= \frac{v^2}{r} = r\omega^2$$



trajectory of motion

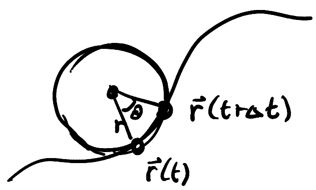
instantaneous radius of curvature for δt

Circular Motion, Newton's 2nd law

Wednesday, February 28, 2007

8:04 AM

Circular Motion



r is the instantaneous radius of curvature r.
As time goes on, you might consider a diff circle

$\theta(t) = 2t^2$ rad

$r = 4m$

a) Find $\vec{v}$ & $\vec{a}$ of a particle in terms of Normal & tangential components at t = 1s

$\omega(t) = \frac{\partial\theta(t)}{\partial t}\Big|_{t=1} = 4t\Big|_{t=1} = 4$ rad/s

$v = r\omega$

$v(1) = 16\,m/s\ \hat{e}_T$

find this w/ $\vec{r}(t)$, find $\hat{e}_N$ w/ $\vec{r}(t)$

$\alpha(t)\Big|_{t=1} = \frac{\partial\omega(t)}{\partial t}\Big|_{t=1} = 4$ rad/s²

$a(t)_T = r\alpha$

$a(t)_T = 16$ rad/s²

$a(t)_N = r\omega^2\ \hat{e}_T$

$a(t)_N = \frac{16^2}{4}\,m/s^2\ \hat{e}_N$

$\vec{r}(t) = r((\cos\theta)t\hat{i} + (\sin\theta)t\hat{j})$

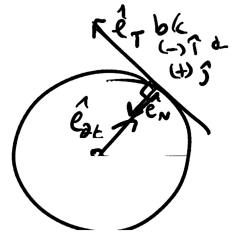
$\vec{r}(t) = r\,\hat{e}_{\theta t}$

$\vec{v}(t) = \frac{\partial\vec{r}(t)}{\partial t} = r\frac{\partial\hat{e}_{\theta t}}{\partial t}$

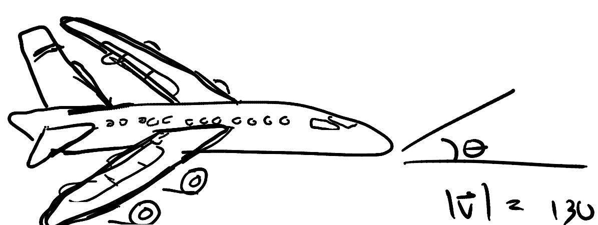
$= r((-\sin\theta)t\frac{\partial\theta(t)}{\partial t}\hat{i} + (\cos\theta)t\frac{\partial\theta(t)}{\partial t}\hat{j})$

$= r\frac{\partial\theta(t)}{\partial t}((-\sin\theta)t\hat{i} + (\cos\theta)t\hat{j})$

$= r\omega(t)\cdot\hat{e}_T$



$\hat{e}_N$ has exact same analysis except instead of v(t), it's $\frac{\partial\vec{v}(t)}{\partial t}$.



$|\vec{v}| = 130$ m/s

Rate of change of path ∡ θ is 5°/s

a) Find acceleration N & T comp.

b) Instantaneous radius of curvature

a) $v = 130\,m/s\ \hat{e}_T$

$a_T = 0$

$a_N = v\omega = 130\left(\frac{5°}{s}\left(\frac{1\,rad}{180°}\right)\right) = 11.3\,m/s^2$

b) $v = r\omega$ or $\frac{v}{\omega} = r$

$a_N = \frac{v^2}{r}$

$r = \frac{v^2}{\ }$ $\quad r \approx 1490\,m$

Now for someone who was smart enough to find his wife is an FBI agent, the author of best-selling books Juniper Sha'aban, Connor

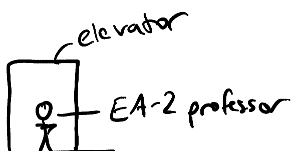
Newton's Second Law

Forces ↔ Acceleration

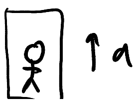
$\sum \vec{F} = \frac{\partial}{\partial t}\vec{p}(t) \leftarrow$ linear momentum $\quad \vec{p}(t) = m(t)\,\vec{v}(t)$

$m(t) = m$

$\sum \vec{F} = m\vec{a}(t)$



$\vec{N} - \vec{W} = 0$



① $\vec{N} - \vec{W} = m\vec{a}$

② $\vec{N} - \vec{W} = 0$

depending on frame of reference.

Use Newtonian frames of reference.

$\sum \vec{F} = m\vec{a}$

$\sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k} = m(a_x \hat{i} + a_y \hat{j} + a_z \hat{k})$

$\sum F_T + \sum F_N = m(a_T \hat{e}_T + a_N \hat{e}_N)$

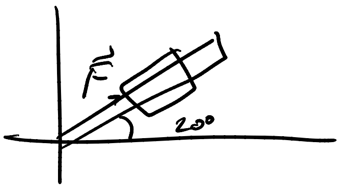
Newton's 2nd law

Friday, March 02, 2007
8:04 AM

$$\Sigma \vec{F}(t) = m\vec{a}(t)$$

$$\Sigma F_x \hat{i} + \Sigma F_y \hat{j} + \Sigma F_z \hat{k} = m(a_x \hat{i} + a_y \hat{j} + a_z \hat{k})$$

$$\Sigma F_T \hat{e}_T + \Sigma F_N \hat{e}_N = m(a_T \hat{e}_T + a_N \hat{e}_N)$$

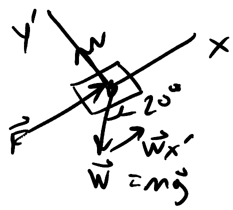


$m = 2\,kg$

$t = 0$ rest

$F = 12\,N$

a) accelleration ∥ to bar



$$F - W_{x'} = m a_{x'}$$

$$12 - mg\sin 20 = m a_{x'}$$

Solve for $a_{x'}$, $a_{x'} = 2.64\ m/s^2$

b) Velocity at $t = 1s$

$$\int_{v(0)}^{v(1)} \partial v = \int_{t=0}^{t=1} a_{x'}\, dt$$

$$v(1) - v(0) = a_x t \Big]_0^1$$

$$v(1) = 2.64\ m/s^2$$

c) Distance traveled?

$$\int_{s(0)}^{s(1)} ds = \int_{t=0}^{t=1} \underbrace{a_{x'} t}_{v}\, dt$$

$$s(1) = \frac{a_{x'} t^2}{2}\Big]_0^1 = 1.32\,m$$

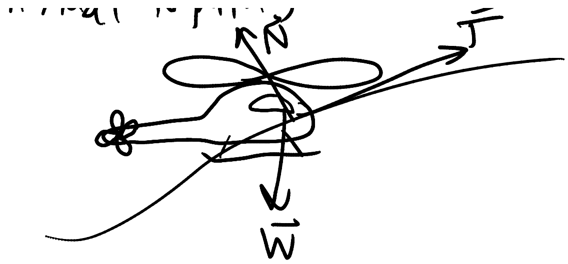
9000 kg helicopter starts from rest at time = 0. $a(t) = .6t\hat{i} + (1.8 - .36t)\hat{j}$ m/s^2, $0 \le t \le 10$

$$a(t) = .6t\hat{i} + (1.8 - .36t)\hat{j} \quad t = 6s?$$

$$\Sigma \vec{F}(t) = m\vec{a}(t)$$

$$\Sigma \vec{F}(6) = 32400\,\hat{i} + 3240\,\hat{j} \quad (N)$$

Express $\Sigma \vec{F}(t)$ in terms of comp. (weight, tangent to path, (T) normal to path (N))



$$\int_{v(0)}^{v(6)} d\vec{v} = \int_{t=0}^{t=6} \vec{a}(t)\,dt$$

$$v(6) - \cancel{v(0)}^{\,0} = .3t^2\,\hat{\imath} + \left(1.8t - .18t^2\right)\hat{\jmath}\,\Big]_0^6$$

$$v(6) = 10.8\,\hat{\imath} + 4.32\,\hat{\jmath}$$

$$\hat{e}_T = .93\,\hat{\imath} + .37\,\hat{\jmath}$$

$$\left[\left(\Sigma \vec{F}(6) - \vec{W}\right) \cdot \hat{e}_T\right]\hat{e}_T = \left[\left(32400\,\hat{\imath} + 85050\,\hat{\jmath}\right) \cdot \hat{e}_T\right]\hat{e}_T$$

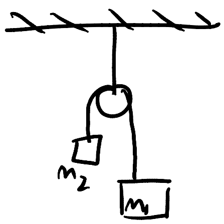
$$\vec{T} = 61600\,\hat{e}_T$$

$$\vec{L} = \Sigma \vec{F}(6) - \vec{W} - \vec{T}$$

$$\vec{L} = -24881\,\hat{\imath} + 62258\,\hat{\jmath} \quad (N)$$

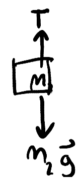
Pulleys, Tension, Newton's 2nd law

Monday, March 05, 2007
8:01 AM

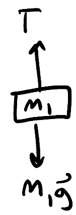


$m_1 = 5\text{kg}$
$m_2 = 2\text{ kg}$

Fixed position, then let go & masses move in m_1's direction.



$\Sigma F = m\vec{a} = T - m_2\vec{g} = m_2\vec{a}_2$



$\Sigma F = ma = T - m_1\vec{g} = m_1\vec{a}_1$

$\vec{a} = \vec{a}_2 = -\vec{a}_1$

Accelerations will be the same, speeds will be the same.

$T - m_2\vec{g} = m_2\vec{a}$ $\quad T - m_1\vec{g} = \cancel{m_1}\vec{a}$

$m_1\vec{g} - T = m_1\vec{a}$

$m_1\vec{g} - m_2\vec{g} = (m_1 + m_2)\vec{a}$

$\vec{g}(m_1 - m_2) = (m_1 + m_2)\vec{a}$

$$\boxed{\frac{\vec{g}(m_1 - m_2)}{(m_1 + m_2)} = \vec{a}}$$

if m_2 was very small, $\vec{a} = \vec{g}$.
if m_1 was very small, $\vec{a} = -\vec{g}$

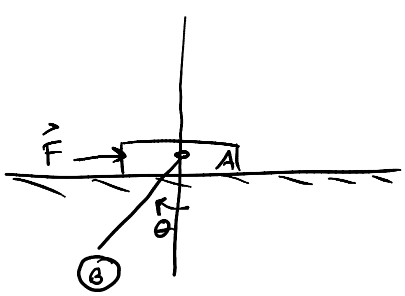
$4.2\,m/s^2 = \vec{a}$

$$\int_{v_0}^{v_f} v\,\partial f = \int_{s_0}^{s_f} a\,ds$$

$v_f^2 - v_0^2 = 2as$

$v_f = \sqrt{2as} = 1.3\,m/s$

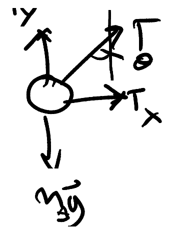
$T = (m_1 g - m_1 a) = 28.05\text{ N}$



$m_A = 30\text{ kg}$
$m_B = 5\text{ kg}$

if $\theta = 20°$ constant, what is $|\vec{F}|$?

$\Sigma F_y = 0 = T_y - m_B\vec{g}$ $\quad \Sigma F_x = m_B a_x$



$\Sigma F_y = 0 = T_y - m_B \vec{g}$

$m_B \vec{g} = T_y$

$m_B \vec{g} = T\cos\theta$

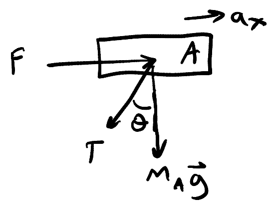
$\frac{m_B \vec{g}}{\cos 20} = T$

$52.2N = T$

$\Sigma F_x = m_B a_x$

$T\sin\theta = m_B \vec{a}_x$

$3.57 m/s^2 = \vec{a}_x$

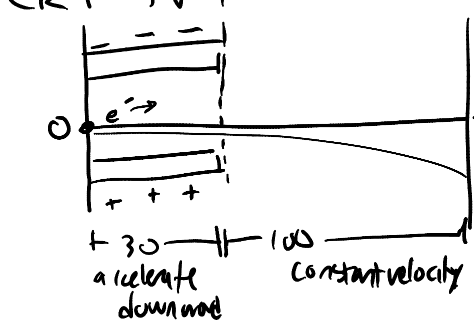


$\Sigma F_x = m_A \vec{a}_x = F - T\sin\theta$

$m_A \vec{a}_x + T\sin\theta = F$

$124.95 N = F$

CRT TV



$m_e = 9.11E-31$ kg

$V_{0x} = 2.2E7$ m/s

$F_y = -eE\hat{j}$ kN

e = charge electron = 1.6E-19 C

E = 15 kN/C

$x(t) = V_x t$

$t = \frac{x(t)}{V_x}$

$t_1 = \frac{30}{2.2E7}$

$t_2 = \frac{100}{2.2E7}$

$t_1 = 1.36E-9s$

$t_2 = 5.9E-9s$

$\int_{y=0}^{y=Y} dy = \int_{t=0}^{t=t_2} v(t)\,dt$

$\int_{t=0}^{t=t_1} v(t)\,dt + \int_{t=t_1}^{t=t_2} v(t)\,dt$

$\Sigma F_y = m_e \vec{a}_y$

$-eE = m_e \vec{a}_y$

$-2.63E15 \, m/s^2 = \vec{a}_y$

Neglect weight of e^- b/c it's really really tiny.

$\int_{v=0}^{v=v(t)} dv = \int_{t=0}^{t=t_1} a\,dt$

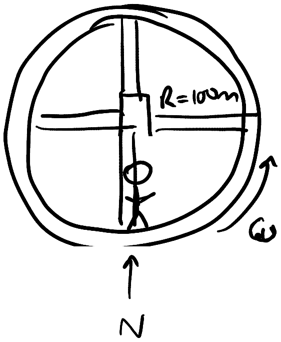
$v(t) = -2.63E15t$

$\int_0^{t_1} -2.63E15\, t\, dt + \int_{t_1}^{t_2} -2.63E15\, t_1\, dt$

$y = -.019m$

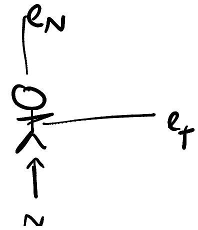
Example Probs (Space Station)

Tuesday, March 06, 2007
8:03 AM



Space station

$N = \frac{mg}{2}$



$\Sigma F \hat{e}_N = m a_N \hat{e}_N$

$N = m a_N$

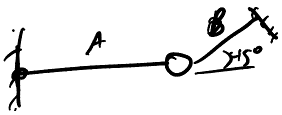
$\frac{mg}{2} = m a_N$

$\frac{g}{2} = a_N$

$\frac{g}{2} = R\omega^2$

$\sqrt{\frac{g}{2R}} = \omega$

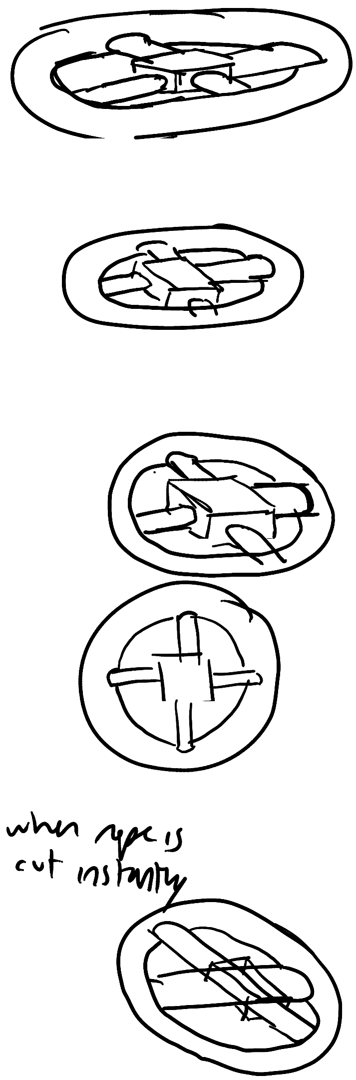
$.2 \text{ rad/s} = \omega$



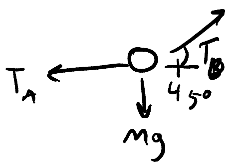
m = 2kg

a) tension A & B?

b) cut rope A, tension B?



a)



$\Sigma F_x = 0$

$T_B \cos 45 - T_A = 0$

$\Sigma F_y = 0$

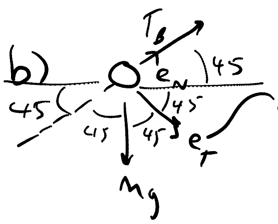
$T_B \sin 45 - mg = 0$

$\frac{mg}{\sin 45} = T_B$

$\boxed{27.75N = T_B}$

$(27.75)\cos 45 = T_A$

$\boxed{19.62N = T_A}$



direction of motion

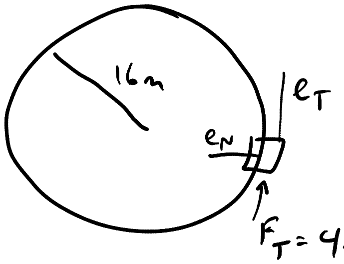
$\Sigma F_N = m a_N$

$T_B - mg\cos 45 = m\frac{v^2}{R}$ ← 0 when rope is cut instantly

$T_B - mg\cos 45 = 0$

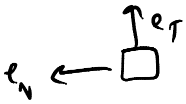
$T_B = mg\cos 45$

$T_B = 13.9N$



$m = 2kg$

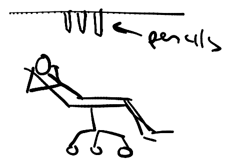
$t=0,\ v_0 = 0$
at $t = 4s$
$v = ?$
Force exerted by bar?



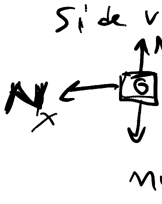
$$\Sigma F_T = m a_T(t) = F_T(t)$$
$$a_T(t) = \frac{4t}{2} = 2t \quad m/s^2$$

$$\int_{v(0)}^{v(4)} dv = \int_{t=0}^{t=4} a_T(t)\,dt$$

$$v(4) - v(0) = t^2 \Big]_0^4$$
$$v(4) = 16 \ m/s$$



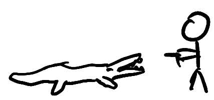
Side view



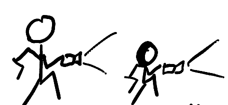
$$\Sigma F_y = 0$$
$$N_y = mg$$
$$\Sigma F_x = m a_N$$
$$N_x = m a_N$$
$$N_x = m\frac{v^2}{R}$$

$$N_x = m\frac{v^2}{R}\Big|_{t=4} = 32N$$

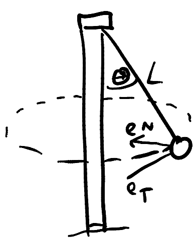
Mulder & Scully run from genetic mutant



Mulder fights an alligator

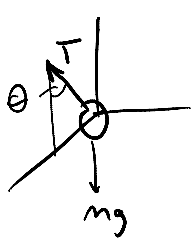


Mulder & Scully w/ flashlights & guns



$m = 1$ slug
$\theta = 30°$
$L = 4ft$

$v = ?$



y direction, x-z plane (N & T)

$$\Sigma F_y = 0$$
$$T\cos\theta = mg$$
$$T = 37.2 \ lbs$$

$$\Sigma F_N = m a_N$$

$$\Sigma F_N = m a_N$$

$$T \sin\theta = m a_N$$

$$\frac{T \sin\theta}{m} = \frac{v^2}{R}$$

$$\sqrt{\frac{R T \sin\theta}{m}} = v$$

$$\sqrt{\frac{4(37.2)\sin 30}{1\,\text{slug}}} = v$$

$$6.1 \ \text{ft/s} = v$$

6 Probs on exam

3D equilib (Find directions of tension of rope or other thing)
- Moments in vector form
- Dot product

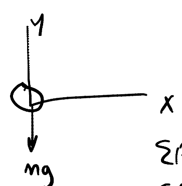
Frame (Internal forces are no longer axial but have directions)
- Separate into members
- Contact forces between members

Mechanics of Materials (Composite rods, 2 materials of diff thicknesses)
- Diff deformations, Hook's Law

Kinematics (2) (Straight-line motion, etc)
- Projectile
- Circular (decompose motion into N & T components)
- Derivations of relations of circular/cartesian.

Dynamics

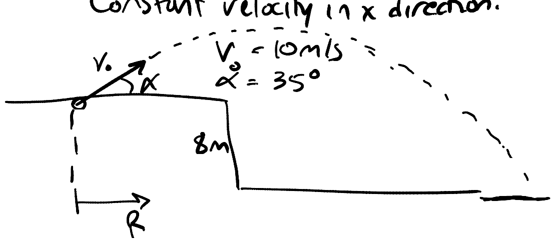
Last 3 are on HW



$$\Sigma F_x = ma_x = 0$$
$$\Sigma F_y = ma_y = mg$$
$$a_y = g$$

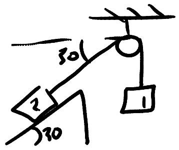
A projectile only has acceleration in y direction.

Constant velocity in x direction.



$$R = (V_0 \cos\alpha)t$$
$$\int_{y=0}^{y=-8} dy = \int_{t=0}^{t=t} v(t)\,dt$$
$$-8 = \int_{t=0}^{t} (v_0 - gt)\,dt$$
$$-8 = v_0 t - \frac{gt^2}{2}\Big]_0^t$$
$$-8 = 10\sin\alpha\, t - \frac{1}{2}gt^2$$
$$R = 16.3\,m$$

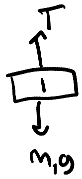
$$\int_{v(0)}^{v(t)} dv = \int_{t=0}^{t} -g\,dt$$
$$v(t) - v(0) = -gt$$
$$v(t) = V_0 - gt$$



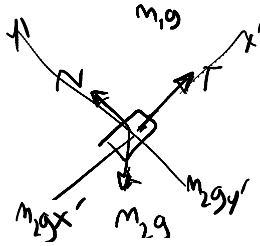
$m_1 = 10$ kg
$m_2 = 5$ kg

a) $\vec{a}_2$ after release
b) $\vec{v}_2$ 2s after release & distance travelled

Velocity & Acceleration of 2 masses are equal



$T - m_1 g = m_1 \vec{a}_1$



$T - m_2 g_{x'}$
$T - m_2 g(\sin 30) = m_2 \vec{a}_2$

BORED
TO
DEATH!

$a_1 = a$
$a_2 = -a$
$T - m_2 g(\sin 30) = -m_2 \vec{a}$
$T - m_1 g = m_1 \vec{a}$
$-m_1 g + m_2 g \sin 30 = m_1 \vec{a} + m_2 \vec{a}$
$\frac{m_2 g \sin 30 - m_1 g}{m_1 + m_2} = a$
$-4.9 \, m/s^2 = a$

$a_2 = 4.9 \, m/s^2$ a_2 is upward.

This means m_1 is accelerating downward

$\int_{v_0}^{v(2)} dv = \int_{t_0}^{t} a_{x'} \, dt$

$v_0 = 0$

$v(2) = a_{x'} t \Big]_{t=0}^{t=2}$
$v(2) = (4.9 \, m/s^2)(2s) = 9.8 \, m/s$

AND THIS CONCLUDES OUR DAILY TORTURE

HAVE A NICE DAY!