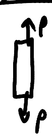
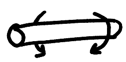
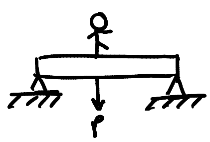
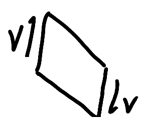
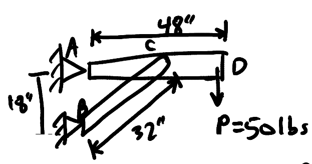
Mechanics of Materials is
a branch of applied mechanics that deals w/ the behaviour of <u>solid bodies</u> subjected to various types of loads.

e.g. shafts, rods, beams, columns

Steps	How
1. What are the forces on the body?	Statics (EA2 Gaaah! Nooo!)
2. what are the stresses?	types of loads, dimension
3. What are the deformations?	Material properties, i.e. E, G, ν, I, L
4. Is it acceptable?	Look at strength, check deflection requirements (serviceability), safety factors

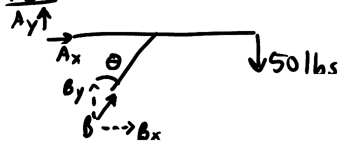
	Types of loads	corres. type of deformation
Axial		 (EA2 Gaaah! Nooo!)
Torque		 θ = rotation cross section
Transverse		1. Bending  2. Shear deformation 

Statics Review Problem



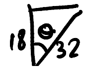
What are the forces exerted on the shelf?

FBD



$B_x = B\sin\theta$

$B_y = B\cos\theta$



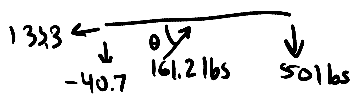
$\cos\theta = \frac{18}{32}$

$\theta = 55.77°$

$\theta = 55.77°$

3 independent unknowns

$\Sigma F_x = 0 = A_x + B_x = A_x + B\sin 55.77 = 0$ (1)
$\Sigma F_y = 0 = A_y + B_y - 50 = A_y + \frac{18}{32}B = 50$ (2)
$\Sigma M_A = 0 = -50(48) + B_x(18) = 0$
$-2400 + (B\sin 55.77)18 = 0$
$\boxed{B = 161.2 \text{ lbs}}$
$A_x + (161.2)\sin 55.77 = 0$
$\boxed{A_x = -133.3 \text{ lbs}}$
$A_y = 50 - \frac{18}{32}(161.2)$
$\boxed{A_y = -40.7 \text{ lbs}}$



Assume that the allowable stress for member BC is $\sigma_{allow} = 30\text{ksi}$
$= 30{,}000 \text{ psi}$

What is the minimum diameter for a circular cross section?

$\sigma_{allow} = \frac{F_{BC}}{A_{min}}$

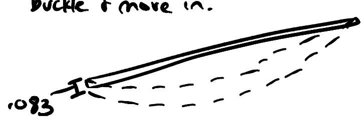
$30{,}000 \text{psi} = \frac{161.2 \text{ lbs}}{A_{min}}$

$A_{min} = 5.37E{-3} \text{ in}^2$

$\frac{\pi d_{min}^2}{4} = 5.37E{-3} \text{ in}^2$

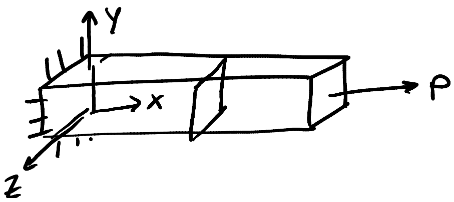
$d_{min} = .083 \text{ in}$

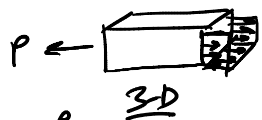
Not a good design b/c it'll buckle & move in.



Office Hrs A153 Fri 12pm-1
Tue 4pm-5

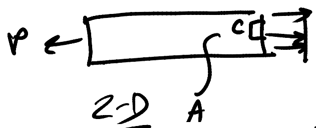
Stresses in axially loaded members





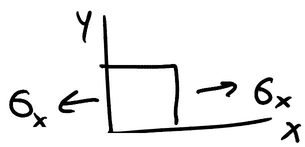
stress resonates over whole x section

$\sigma_x = \frac{P}{A}$

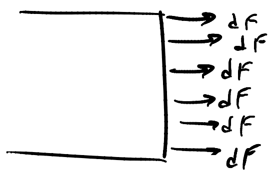


$\sigma_x = \frac{P}{A}$

$\sigma = \frac{F}{A}$ in general

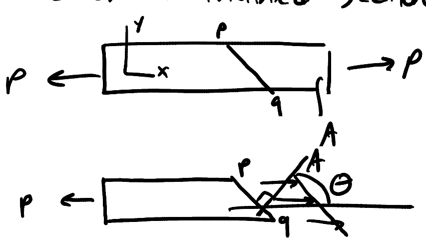


stress-element C

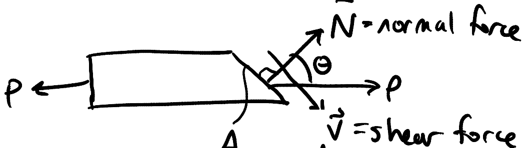


$$\int dF = \int_A \sigma \, dA = P$$

Look at inclined section for a more complete picture



θ is defined between x axis & plane normal to inclined section

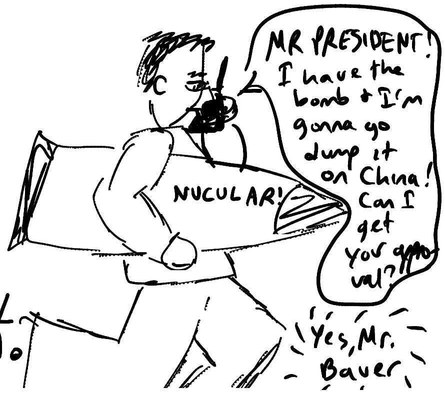


$A_\theta = \frac{A}{\cos\theta}$

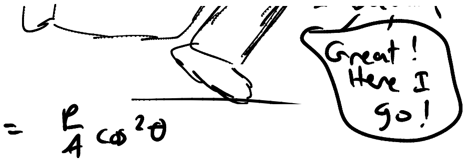
$$\boxed{\begin{aligned} N &= P\cos\theta \\ V &= P\sin\theta \end{aligned}}$$

$\sigma_\theta = \frac{N}{A_\theta}$ Normal

$\tau_\theta = \frac{V}{A_\theta}$ shear



Stress Stress

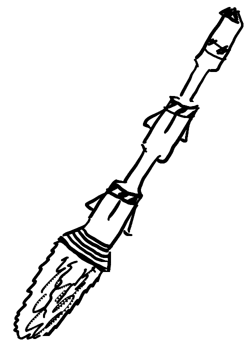


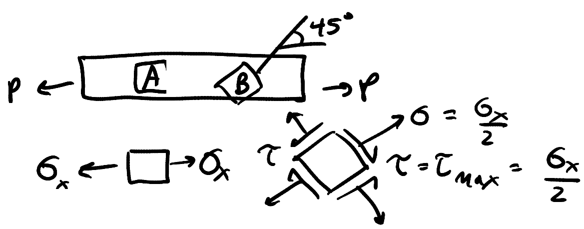
substitute $\sigma_\theta = \dfrac{P\cos\theta}{\left(\frac{A}{\cos\theta}\right)} = \dfrac{P}{A}\cos^2\theta$

$$\boxed{\sigma_\theta = \sigma_x \cos^2\theta}$$

substitute $\tau_\theta = \dfrac{P\sin\theta}{\left(\frac{A}{\cos\theta}\right)} = \dfrac{P}{A}\sin\theta\cos\theta$

$$\boxed{\tau_\theta = \sigma_x \sin\theta\cos\theta}$$



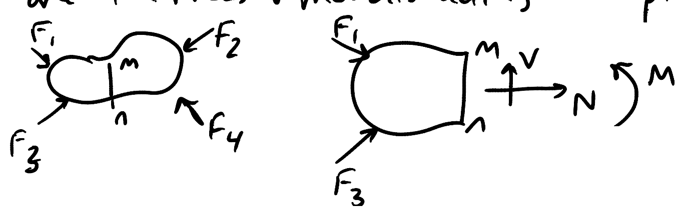


$\frac{\sigma_x}{2}$ b/c σ_x in either direction.

$$\frac{\sigma_x}{2} + \frac{\sigma_x}{2} = \sigma_x$$

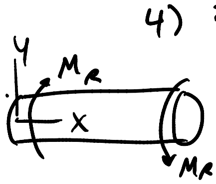
Internal resultant loading

What are the forces & moments acting on a pt inside an object?

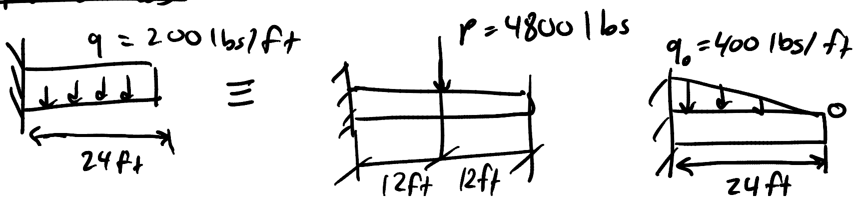


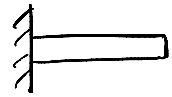
Four diff types of internal resultant loads
1) $\vec{N}$ = normal force (normal to cut section)
2) $\vec{V}$ = shear force (parallel to cut section)
3) $\vec{M}$ = bending moment (about axis ∥ to cut section)
4) $\vec{T}$ = torque (about axis normal to section)

ex.



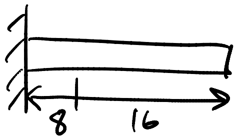
Distributed loads

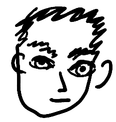


$\equiv$ 

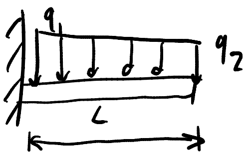
$P = \frac{1}{2}(24ft)(400\,lbs/ft) = 4800\,lbs$

Centroid is $\frac{1}{3}$ from the base. So in this case 8 ft from wall

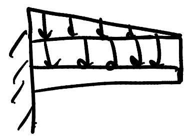


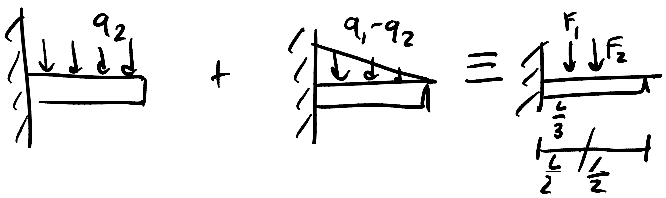


Trapezoidal load



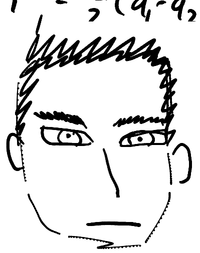
load is rectangle w/ triangle on top.



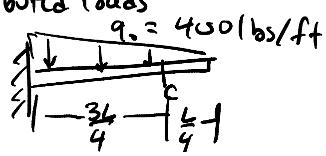


$F_2 = q_2 L$

$F_1 = \frac{1}{2}(q_1 - q_2)L$



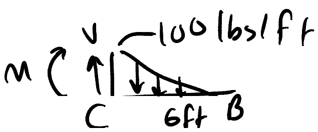
<u>Ex</u> FBD of cut section involving distributed loads



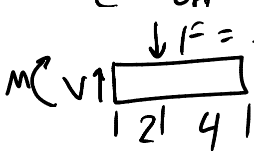
what is resultant internal load at C?

given $L = 24$ ft

100 lbs/ft



$\downarrow F = \frac{1}{2}(6ft)(100\,lbs/ft) = 300\,lbs$



$\Sigma F_y = 0$

$\Rightarrow \boxed{V = 300\,lbs}$

$\Sigma M_c = 0$

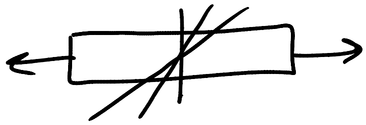
$\Sigma M_c = 0$

$-300(2\,ft) - M = 0$

$\Rightarrow \boxed{M = -600\ lbs\text{-}ft}$

Stresses, distributed loads

Friday, September 28, 2007
9:59 AM

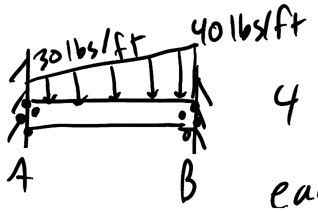


Normal stress
Shear stress (parallel to plane we're considering)

∡ of plane w/ greatest shear stress is 45°

proffessor Jennings A131-4(?)

1-199)



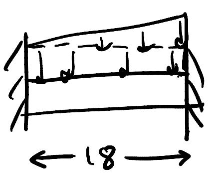
4 bolts holding up bar

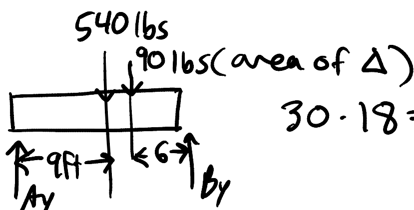
each nail has diameter of 0.25 in

$d = 0.25$ in

Simplify distributed load to single force acting at a pt or to ~~t~~two forces

You place the force at the centroid





$30 \cdot 18 = 540$ (area of ▭)

Centroid of the Δ is $\frac{1}{3}$ from the base to the top

Clockwise (-)
Counterclockwise (+)

$$\Sigma M_A = 0 = F_B(18) - (540)9 - 90(12)$$

$$F_B = 330 \text{ lbs}$$

$$F_A = +330 - 540 - 90 = 300 \text{ lbs}$$

$$\sigma_s = \frac{F}{A} = \frac{300}{4(\pi r^2)} - \frac{300}{4(\pi(\frac{.25}{2})^2)} = 1.53 \text{ ksi}$$

other side too, & you get 1.68 ksi

Determine the smallest diameter for the nails. σ allowable is 4 ksi

$$\sigma = \frac{F}{A} = \longrightarrow 4(10^3) = \frac{300}{4\pi r^2}$$

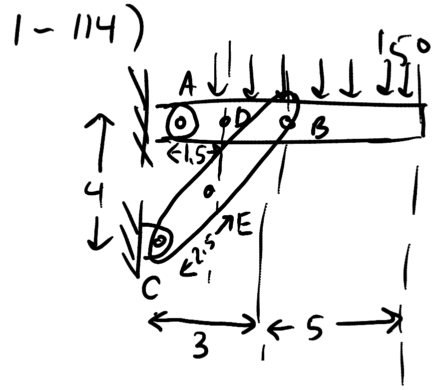
ksi

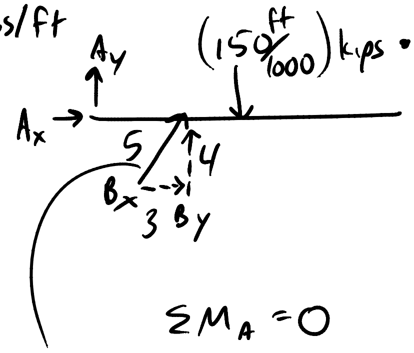
$d = .155\text{ in}$

do the same thing for F_B. $d = 0.162\text{ in}$

42! The answer to life, the universe, and everything!

1-114)





$\left(150 \frac{ft}{1000}\right)$ kips $\cdot$ 8 ft = 1.2 kips

2 kips

$\Sigma M_A = 0$

$= 3(B_y) - 1.2(4)$

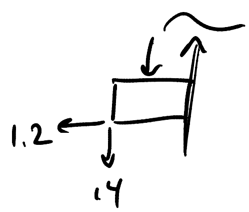
$B_y = 1.6$ kips

$\Sigma M_B = 0 = -1(1.2) + 3A_y$

$A_y = .4$ kip

$A_x = -1.2$ kip

Force at D?



$V_D - .225 - .4 = 0$

$V_D = .625$

$\Sigma M_D = 0 \quad 0 = \frac{1}{2}(1.5) \times V_D + .4(1.5) + M_D$

$M_D = -.769$ kip$\cdot$ft

Strain (normal, shear)

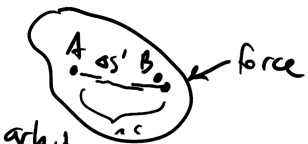
Monday, October 01, 2007

10:00 AM

Define <u>Strain</u> - no units Characterizes the deformation of a material

All things deform under a load.

ε_n = normal strain = $\lim_{\substack{B \to A \\ \text{along line}}} \frac{\Delta s' - \Delta s}{\Delta s} \approx \frac{\Delta L}{L}$



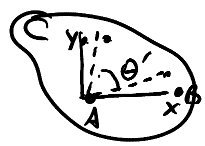
arbitrary object

$\Delta s'$ after force

Δs before force

if negative length it gets shorter.

If positive it's getting longer



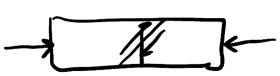
axes change in length

angle between them changes. θ'

Shear strain ← radians

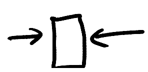
γ = Change in $\frac{\pi}{2}$

$\frac{\pi}{2} - \lim_{\substack{B \to A \\ C \to A}} \theta'$ $+ < \frac{\pi}{2} < -$

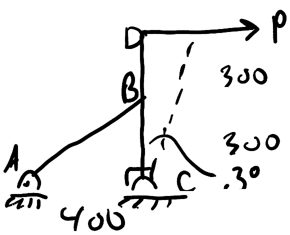


That kind ⫽ of stress causes shear strain.

The 90° ∡ is changing to some other ∡



no strain in this direction. Just squishes.



Normal strain in cable AB

CBD is rigid

$\varepsilon_n = \frac{\Delta L}{L}$ AB = 500 mm

AB'

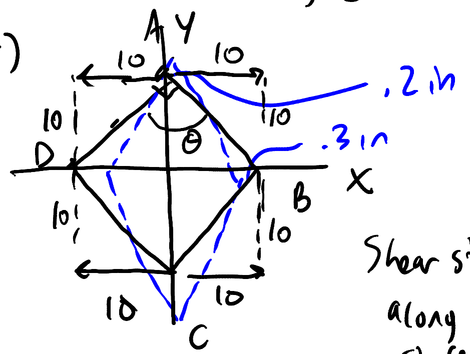
law of cosines $a^2 = b^2 + c^2 - 2bc\cos A$

$$(AB') = \sqrt{400^2 + 300^2 - 2(400)(300)\cos(90.3)}$$

$$AB' = 501.255 \text{ mm}$$

$$\varepsilon_n = \frac{1.255}{500} = \text{some \#}$$

2.15)



$$\varepsilon_x = \frac{-.3}{10} = -.03 \quad \text{it's getting shorter}$$

$$\varepsilon_y = \frac{.2}{10} = .02$$

Shear strain at A, along AB, other stuff like that

$$\frac{\theta'}{2} = \tan^{-1}\left(\frac{9.7}{10.2}\right) = 43.561°$$

$$\theta' = 1.52056 \text{ rad}$$

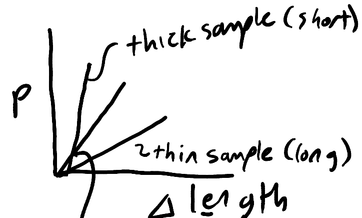
$$\gamma_A = \frac{\pi}{2} - \theta'$$

$$\gamma_A = \frac{\pi}{2} - 1.52056$$

$$\boxed{\gamma_A = 0.0502 \text{ rad}}$$

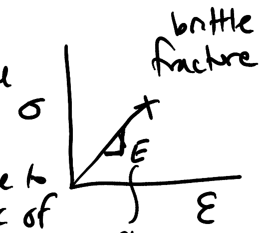
To test strength, put loads on it.

We measure the force needed to pull the material apart to the point where it breaks or won't go back to its original place.



Obey's Hooke's Law → Linear Elastic Material

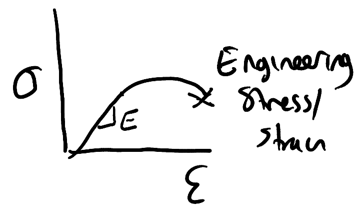
Normalizes the curve b/c it isn't susceptible to change b/c of short, long, thin, thick samples



Slope, Young's Modulus

Chalk, Glass

No permanent deformation

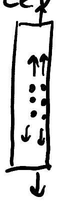


Engineering Stress/Strain

Covalent bonding is very directional — stretch then break b/c of the covalent bonding. Glass is very strong but very brittle

Slip happens w/ non brittle stuff (steel, copper), which is shear stress

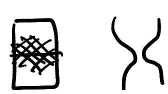
Atom Level



Atoms separate

1 of 2 things happens

- Atoms stretched far enough so that the bonds break (fracture)
- Atoms slip & form a neck.

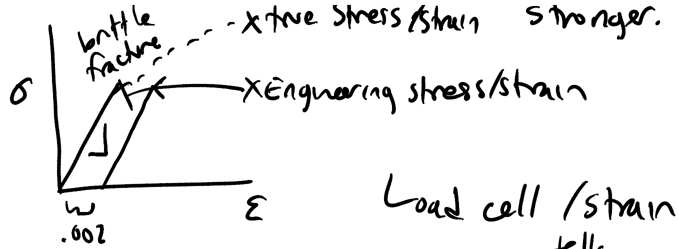


plastic deformation

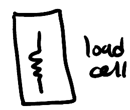
The incremental stress $= \dfrac{F}{\text{cross section}}$ ← gets smaller

Strain hardening — when strain makes a material stronger.

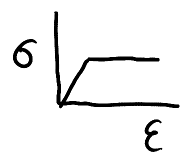
brittle - - x true stress/strain



Load cell / strain gauge
tells you how far it stretches based on how far spring goes.

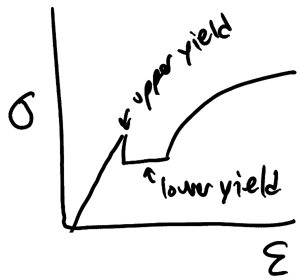


I WANT TO BELIEVE



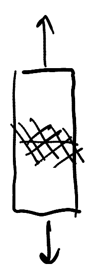
Ideal elastic/plastic

Miles (?) Steel is Steel w/ some Carbon in it



yield pt = plastic deformation beginning

Sliding planes of atoms against one another. Plastic deformation is always a consequence of shear stress.



dislocations - discontinuities in slip planes

Instead of forcing the paper to slide, small events happen at the same time.

This never happens w/ a pure material - only w/ a material w/ stuff mixed.

Dislocations get stuck → atoms repel or attract & get "favorite" spots.

Upper yield is there b/c mechanism of slip isn't immediately available to it. Ideal plastic flow, then engineering stress/strain.

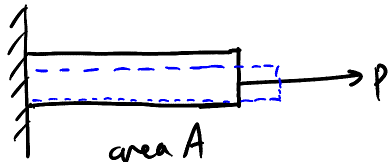
CSI:MIAMI

Superplastic was discovered by a guy who wanted to get a drink so put the machine on slow & it stretched

Poisson's Ratio, Shear Strain

Wednesday, October 03, 2007
10:02 AM

Poisson's ratio $\quad \nu$



Acts like a linear elastic spring as it is pulled

area A

$\sigma = E\varepsilon$

$\varepsilon_x = \frac{P}{AE}$

A rubber band is very far from being linear elastic.
A billiard ball is very close to being linear elastic.
Materials close to being perfectly linear elastic don't deform much when impacted or pulled at.

Isotropic material

$$\nu = \frac{\varepsilon_{lat}}{\varepsilon_{long}} = \text{Poisson's ratio}$$

$$-\nu\,\varepsilon_x = \varepsilon_y = \varepsilon_z$$

$$.25 < \nu < .33 \qquad \nu_{max} = .5$$

Final Volume $\rightarrow$ $V_f = (1+\varepsilon_y)(1+\varepsilon_x)(1+\varepsilon_z)$

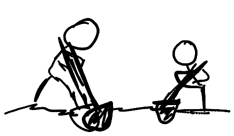
$$V_f = (1+\varepsilon_x)(1-\nu\varepsilon_x)^2$$

$$(1+\varepsilon_x)(1-2\nu\varepsilon_x + \cancel{\nu^2\varepsilon_x^2})$$ small

$$\varepsilon_x \approx 2\nu\varepsilon_x$$

$$\nu \approx .5$$

$$V_i = 1$$

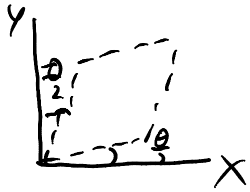


"Dig."

If a const vol material, a liquid, then it's constant .5
If a tensile load changes the volume (squash it), then it gets different & all the signs change.

Young's Modulus is a spring constant, like k.
Poisson's Ratio is a proportionality constant between normal stress & normal strain

Shear stress & shear strain?

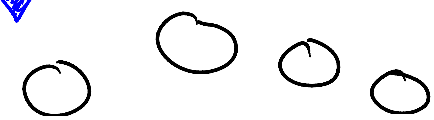


$$\underbrace{\gamma_{yx}}_{\text{shear strain}} = \gamma_{xy} = \frac{\pi}{2} - \theta$$

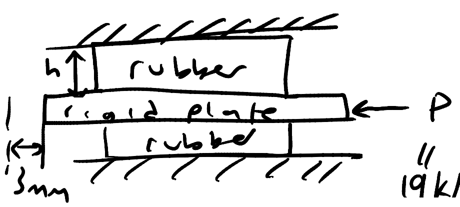
$$\tau_{xy} = \underbrace{G}_{\text{shear modulus, constant}} \gamma$$

$$\boxed{\frac{E}{2G} = 1 - \nu}$$

Important!



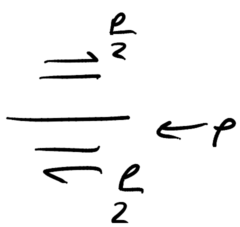
There Are 4 lights!

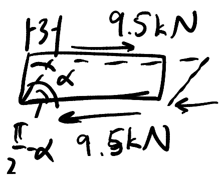


$P = 19\,kN$ $w = 60\,mm$ $h = 35\,mm$

$L = 180\,mm$

Shear modulus of rubber?





$$\tau = \frac{9.5\,kN}{(180)(60)} = 879.6\,kPa$$

$\alpha = .08571\,\text{rad}$ $\tan^{-1}\left(\frac{3}{35}\right) \sim \frac{3}{35}$

$\frac{\pi}{2} - \alpha$ = strain γ

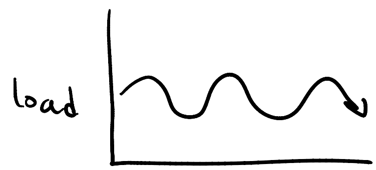
$\tau = G \cdot \gamma$

$G = 10.26\ MPa$

Creep - slow sagging

or materials, deformation
Creep occurs in really
slow increments.

Fatigue - failure/damage due to
continuous cyclic loading

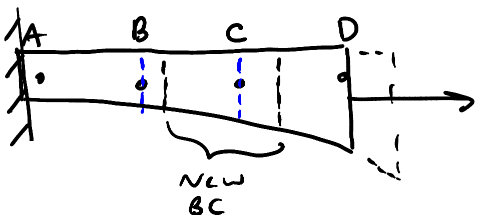


bending a paper clip over & over
again
Crank shaft in car fails, at
150,000 mi
Airplanes' wings

Force, Stress, Displacement

Friday, October 05, 2007
10:03 AM

σ = stress (normal to plane) Tensile & compression
ε = strain
δ = displacement of some kind
ν = Poisson's Ratio $\frac{\varepsilon_{lat}}{\varepsilon_{long}}$
γ = shear strain
λ = shear stress



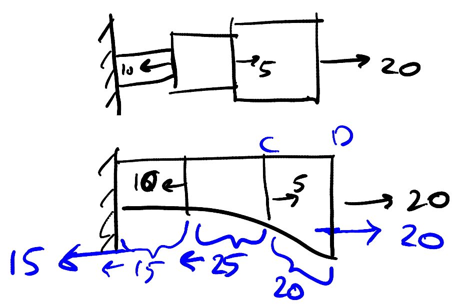
what is displacement BC, δ_{BC}

$$\sigma = \frac{P(x)}{A(x)} \qquad \varepsilon = \frac{d\delta}{dx}$$

$$\sigma = E\varepsilon$$

$$\frac{P(x)}{A(x)} = E\frac{d\delta}{dx}$$

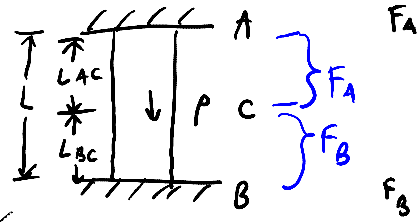
$$\boxed{\delta = \int \frac{P(x)}{} \, dx}$$



Make a cut between C & D & you see that it's 20

Positive b/c it's in tension.
20 to the right

Indeterminant problem is a prob w/ more unknowns than equations.



$F_A + F_B - P = 0$

We know P but not F_A or F_B

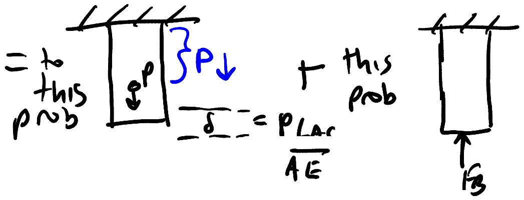
$\delta_{AC} - \delta_{BC} = 0$

Superposition
Force
Displacement

$$\frac{F_A L_{AC}}{AE} - \frac{F_B L_{BC}}{AE} = 0$$

$$F_A = P\left(\frac{L_{BC}}{L}\right)$$

$$F_B = P\left(\frac{L_{AC}}{L}\right)$$

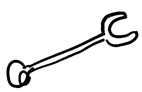


We can compute F_B if we know A, E, & L

Force between A & C changes by removing one end.

$$F_B = \frac{\delta AE}{L}$$

↑ original length



24

THE FOLLOWING TAKES PLACE BETWEEN 11:00AM AND 12:00 PM ON FRIDAY!!!

1144

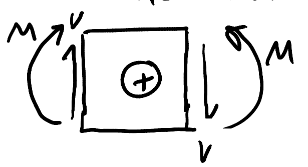
Torsion, Internal Forces, Stress Concentration

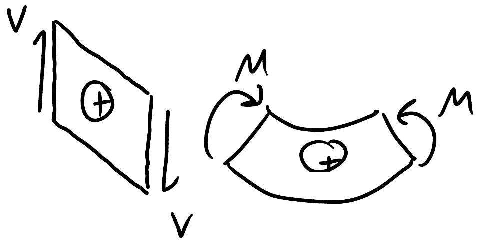
Monday, October 08, 2007
9:59 AM

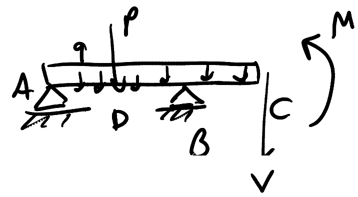
Deformation sign convention

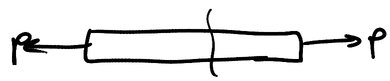
HOW LONG CAN WE DELAY CLASS?

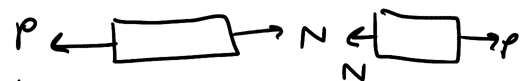
Slice of a beam







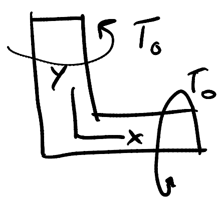




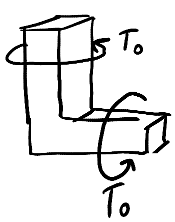
$\Sigma F_x = -N + P = 0$

static sign conventions

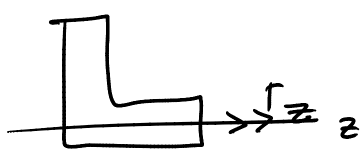
Torsion is pos when acting on direction of axis you're looking at.



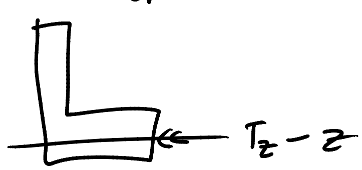
right hand rule.

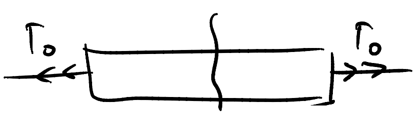


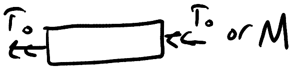
or



or







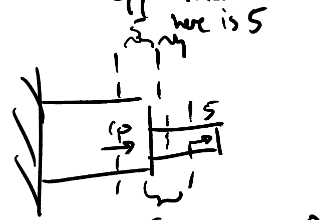
$\Sigma M_x = 0$

$-T_0 - M = 0$

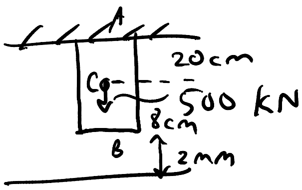
$M = -T_0$

Internal Forces are forces that cause deformation.

Defined by couples. Either stretching or squishing material. Bending material is result of application of 2 = & opp moments.

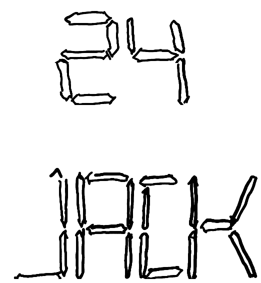


Force here is 0.



$A = 10 \text{ cm}^2$

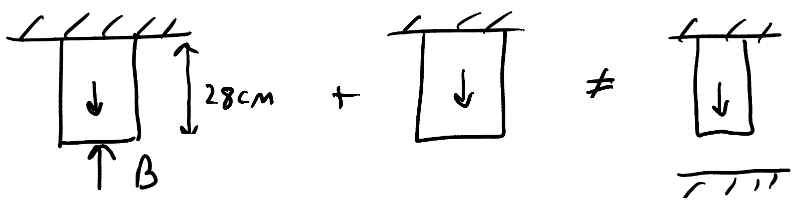
$E = 25 \text{ GPa}$

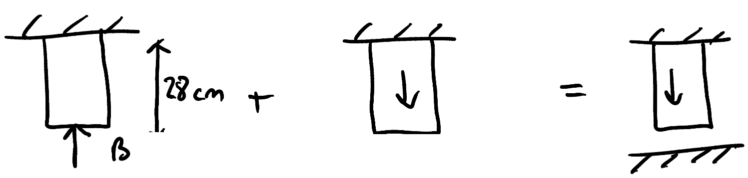


See is 500kN will close gap.

$$\delta_{A/B} = \delta_{AC} + \cancel{\delta_{CB}}^{\,0}$$

$$= \frac{PL}{AE} = \frac{(500E3)(.2m)}{(25E9Pa)(10E-4\ m^2)} = 4\text{ mm}$$





Force at B

$$\delta_{AB} = -2\text{ mm} = \frac{R_B\,(.28m)}{AE} =$$

$R_B = 178.6 \text{ kN} \uparrow$

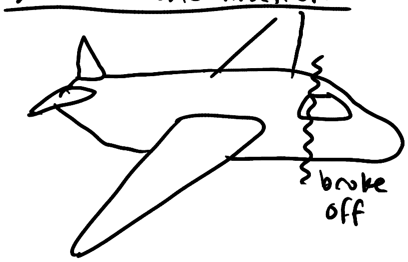
$R_A = 321.4 \text{ kN}$

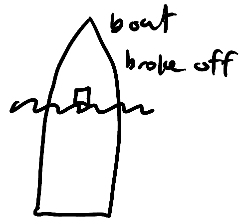
1133

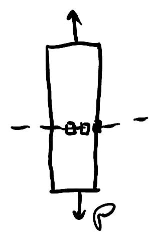
After gap is closed, B pushes up, compresses rod a

After gap is closed, B pushes up, compresses rod a bit.

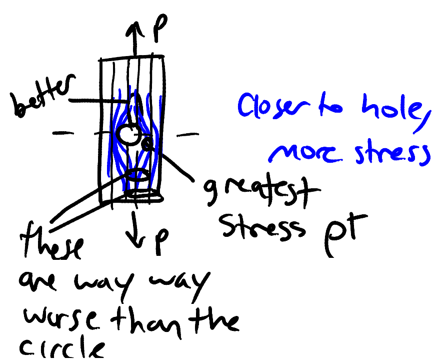
Stress concentration







$\sigma = \frac{P}{A}$



Oval shape is better than square shape b/c stress concentrators are less.

1948, stuff changed & they started using ovals, not squares.

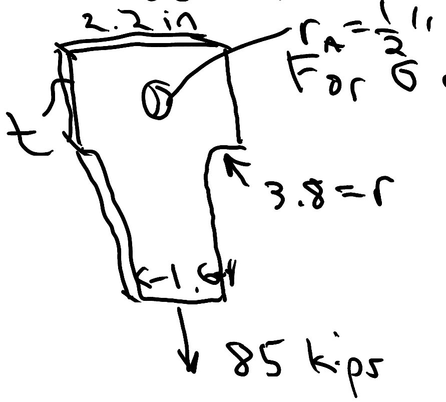
p165

Holes, Torsion

Tuesday, October 09, 2007

10:03 AM

Woohoo!

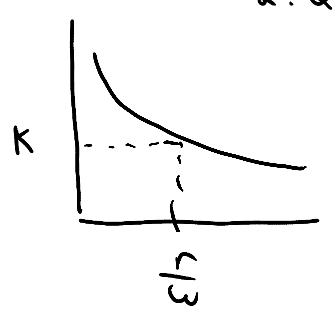


For σ allowed = d = 18 ksi
What is min thickness t?

hole $\sigma_{Avg} = \frac{F}{A_{min}} = \frac{8.5}{(2.2-1)t} = \frac{7.083}{t}$

Area of load bearing capability of cross section.

$$\frac{r}{w} = \frac{.5}{2.2} = 0.227$$



$K \approx 2.4$

$$\boxed{K = \frac{\sigma_{max}}{\sigma_{Avg}}}$$

IMPORTANT!

$$K\sigma_{Avg} = \sigma_{max}$$

$$2.4\left(\frac{7.083}{t}\right) = \sigma_{max}$$

$$\frac{17.0}{t}\text{ ksi} = \sigma_{max}$$

So use the function to determine the minimum val of t.

Fillets –

$$\sigma_{max} = \frac{P}{A_{min}} = \frac{18.5}{1.6t} = \frac{5.313}{t}$$

$$\frac{w}{h} = \frac{2.2}{1.6} = 1.375$$

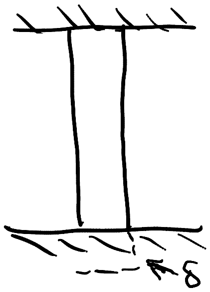
$$\frac{r}{h} = \frac{\left(\frac{3}{8}\right)}{1.6} =$$

$$\sigma_{max} = \frac{8.5}{t}$$ (Same logic as before)

The hole is worse for stress concentration

$t \geq .944$ $\quad$ $(17 > 8.5)$

The hole is always the worst case, or a $\square$.



No stress at rm temp
No forces to start, rxns = 0

Heat to a higher temp, compute force?

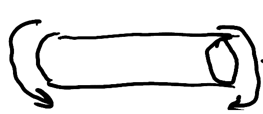
Must compute expansion.

$$\delta_T = \alpha(\Delta T)L$$ ← length.

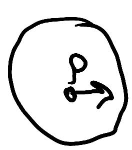
δ_T ↑ displacement; α ↑ const; ΔT ← Δ temp

Compute force to push back into restraint like other problems.

<u>Torsion</u>



T_1 T_2 Equal & opposite torque



Magnified version of the end

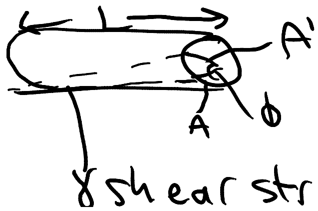
1/31 END

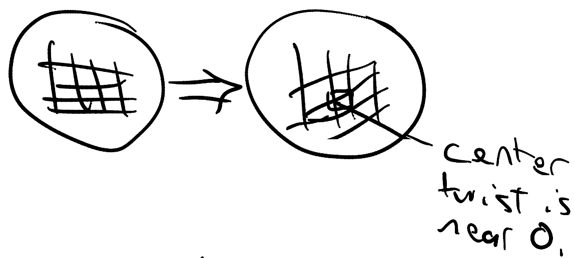
THE

$$\int \rho \, dF = T$$

$\sigma = \frac{P}{A}$

T is external force but it isn't uniformly distributed





close approx: $\tan\gamma = \frac{AA'}{L} \approx \gamma$

$\phi = \frac{AA'}{\rho}$ ← can vary from the center out

$$\gamma = \frac{\rho \phi}{L}$$

L = length of bar
ρ = radial dist from center
ϕ = overall ∡ of twist

Shear strain ↑ w/ ∡ of twist
↓ w/ L of sample

$\tau = G\gamma$

integrate to get torque

$$\int \rho \, dF = T$$

HW
$V = \rho \cos\theta$
$\theta = 0$
$dF = P$

Torque, Torsion

Wednesday, October 10, 2007
10:00 AM

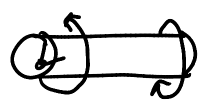
Torsion = moment about an axis

$$G\gamma = \frac{\rho}{c} G\gamma_{max}$$

ρ ← diameter @ pt we analyze

G ↑ shear modulus, γ ↑ shear strain

ϕ = ∠ of twist



If we integrate from center, where there is no stress or strain, ~~are~~ to the outside, we get ∠ of twist.

$$\int \rho(\lambda\, dA) = T \leftarrow \text{torque}$$

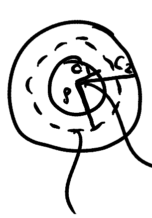
Integrate strain, get ∠ of twist
Integrate stress, get applied torque

$$T \to \frac{\tau_{max}}{c}\int_0^{c_{max}} \rho^2\, dA \leftarrow \text{moment of inertia} = J$$

$$T = \frac{\tau_{max} J}{c}$$

Shear stress @ any pt $= \frac{T\rho}{J}$

or if $\lambda_{max} = \tau_{max} = \frac{Tc}{J}$



A wheel has spokes to make it lighter.

mass doesn't do anything.

dA

$$J = \int \rho^2 dA = \int 2\pi\rho^3 d\rho = \frac{2\pi\rho^4}{4}\Big]_{c_1}^{c_2}$$

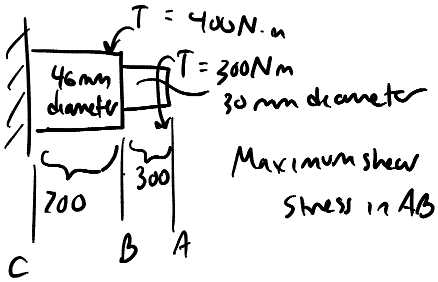
c_1 = inside
c_2 = outside
$dA = \pi r^2$

$(2\pi\rho\, d\rho)$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$$ ← Very useful equation

$c_1 = 0$? then $J = \frac{\pi}{2} c_2^4$ b/c there is no inside.

$J \uparrow$.



Maximum shear stress in AB & BC?

Jack Bauer vs. Fox Mulder

Maximum torque is not dependent on the length.

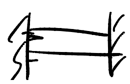
Something that's long ~~TB~~ will deform more than something that's short.

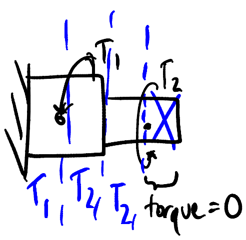
$$\tau_{max} = \frac{Tc}{J}$$ — c is radius

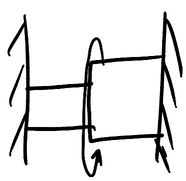
$J = \frac{\pi}{2}(c^4)$

$$\tau_{max\,AB} = 56.6\,MPa$$

$$\tau_{max\,BC} = 36.6\,MPa$$



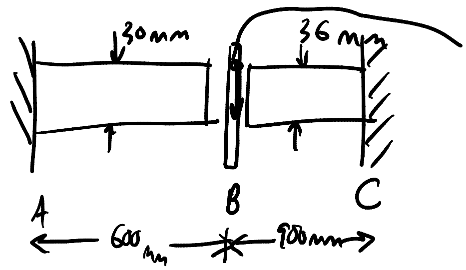




We have to use info about dispacement to get to the answer.

Maximum Shear Stress

Friday, October 12, 2007
9:58 AM



$T = 500\ N \cdot m$

Maximum shear stress in 2 shafts

Indeterminate problem, so compatibility condition we will use is there is no net twist. ∡ twist in 1 shaft is ∡ twist in 2nd shaft

Max shear stress is in surfaces of each of the 2 shafts. 2 separate problems here. Torsion in each.

$$\lambda_{max} = \tau_{max} = \frac{TC}{J}$$

C ← radius

J ← moment of inertia

This problem has 1 dimension. The only thing applied is a torque.

Can't solve by statics b/c we have only 1 dimension so 1 eqn, but more unknowns.

$\Sigma F = 0$

$\Sigma M = 0$

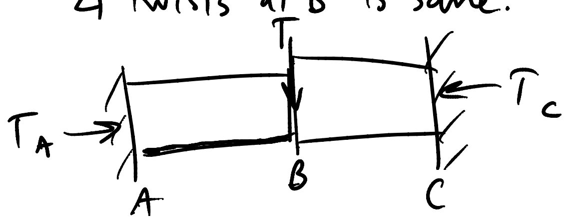
3 equations for each, 3 unknowns total.

$\Sigma F_x, \Sigma F_y, \Sigma F_z$

$\Sigma M_x, \Sigma M_y, \Sigma M_z$

If we had 2 dimensions (eqns) for each including moments we've got 3 unknowns.

∡ twists at B is same.



We need 2 eqns.

$$T_A + T_B + T_C = 0$$

$\phi_{AB} = \phi_{BC}$ Compatiblity condition

$C_{AB} = .015m$

$C_{CB} = .018m$

$J_{AB} = \frac{\pi}{2}(.015)^4 = 7.952E-8\ m^4$

$J_{CB} = \frac{\pi}{2}(.018)^4 = 16.490E-8\ m^4$

$\phi_{AB} = \phi_{BC}$

$$\boxed{\frac{T_A L_{AB}}{J_{AB} \cancel{G}} = \frac{T_C L_{BC}}{J_{BC} \cancel{G}}}$$

$$\frac{T_A L_{AB}}{J_{AB}} = \frac{T_C L_{BC}}{J_{BC}}$$

$T_A + 500Nm + T_C = 0$

$T_A = 290.1\ Nm$

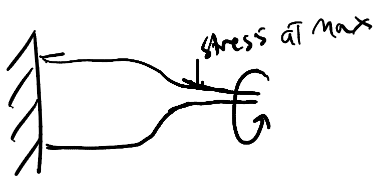
$T_C = 209.9\ Nm$

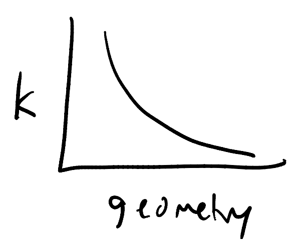
Shear stress max in AB is

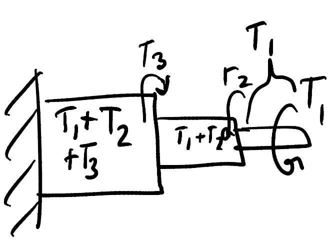
$\tau_{\substack{max\\AB}} = \frac{T_A C}{J} = 39.6\ MPa$

$MPa = Pa\,E6$

$\tau_{\substack{Max\\BC}} = \frac{T_C C}{J} = 31.7\ MPa$







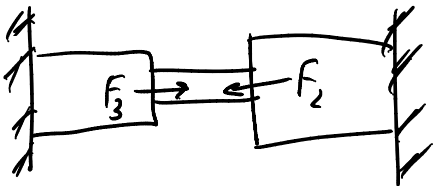
Determinate. 1 Dimension.

Dynamic issue of Power.

$$\boxed{P = T\omega}$$ ← angular velocity rad/sec

watts = (N·m)(rad/s)

ft lb/s = "



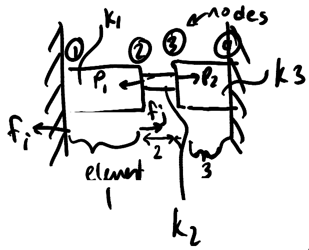
2 unknowns

2 compatibility requirements → whole thing = 0.

Matrices, Bending & Moments

Monday, October 15, 2007

9:01 AM



Element 1

spring const.

$$\begin{Bmatrix} f_i^1 \\ f_j^1 \end{Bmatrix}_{internal} = \begin{Bmatrix} d_i^1 \\ d_j^1 \end{Bmatrix} \begin{bmatrix} k_{11}^1 & k_{12}^1 \\ k_{21}^1 & k_{22}^1 \end{bmatrix}$$

Element 2

$$\begin{Bmatrix} f_i^2 \\ f_j^2 \end{Bmatrix}_{int} = \begin{Bmatrix} d_i^2 \\ d_j^2 \end{Bmatrix} \begin{bmatrix} k^2 & k^2 \\ k^2 & k^2 \end{bmatrix}$$

These are internal forces, not external P_1 + P_2

most important thing →

$$\boxed{\begin{aligned} \Sigma F &= 0 \\ f^{int} - f^{ext} &= 0 \end{aligned}}$$

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Bmatrix}_{ext} = \left(\begin{bmatrix} k' & k' & 0 & 0 \\ k' & k' & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k^2 & k^2 & 0 \\ 0 & k^2 & k^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k^3 & k^3 \\ 0 & 0 & k^3 & k^3 \end{bmatrix} \right) \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix}$$

Unknowns

$d_1 + d_4 = 0$ b/c of restraints at end!

$$f_1 = k'\cancelto{0}{d_1} + k'd_2$$
$$f_1 = k'd_2$$

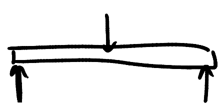
$f_2 = P_1$

$f_3 = P_2$

$$\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{bmatrix} k'+k^2 & k^2 \\ k^2 & k^2+k^3 \end{bmatrix} \begin{Bmatrix} d_2 \\ d_3 \end{Bmatrix}$$

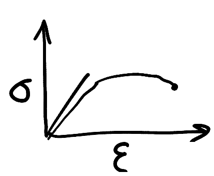
You can do the same thing in finite elmts. Instead of a force, put a torque, instead of displacement it's ∡ of twist, & instead of k it's a shear spring constant

__Bending & Moments__



3 pt bend test

THE X FILES

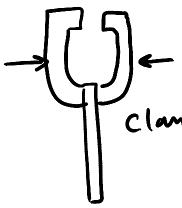


__Yield strength__ - how strong it is till permanent deformation.

We pull it apart in tension b/c it can pull apart by normal stress.

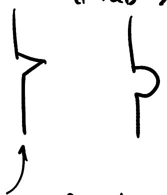
Shear causes sliding, plastic deformation.
Normal stress causes failure.

Becomes difficult to bend with brittle materials
Does not plastically deform.



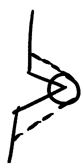
clamp being squished, create stress concentration.

Brittle materials susceptible to stress concentration.



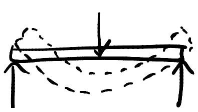
worse for stress concentration.

Metals can deform to tolerate stress.



Make a circle then you get rid of the concentration

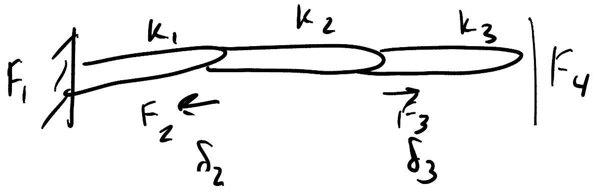
Brittleness + wear resistance go together.



Forces, Torque, Moments

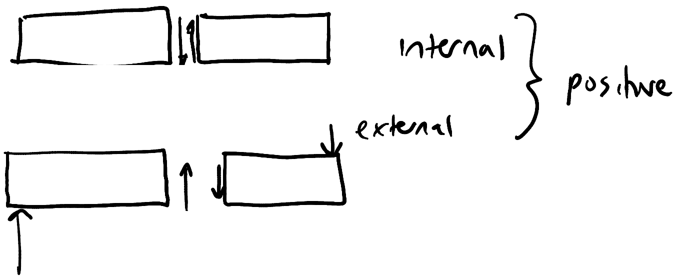
Tuesday, October 16, 2007

10:03 AM



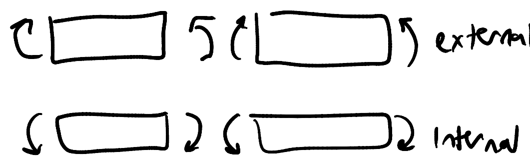
$F^{int} = k_1 \delta_2$ $\qquad$ $\Sigma F_{int} = \Sigma F_{ext}$

Sign



$\Sigma F^{int} = \Sigma F^{ext}$

$\Sigma M^{int} = \Sigma M^{ext}$



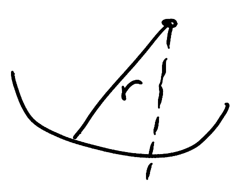
internal is opposite to the external

Clockwise in torque $= (+)$

Clockwise in Moments $= (-)$

bent like this means $(+)$

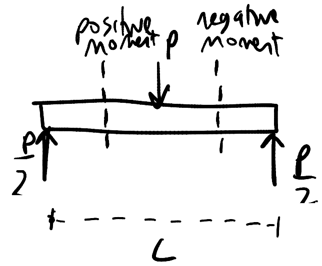
pure bending causes

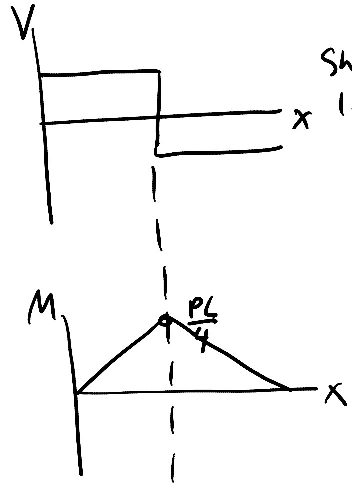


circle.

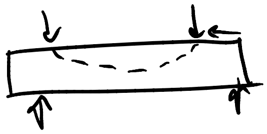
$M = (\vec{r} \times \vec{F})$

if this bends around pos. axis, it's positive.

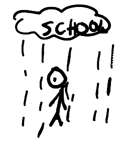


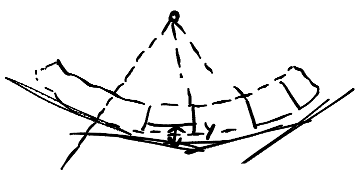


Shear force
Internal along the length.

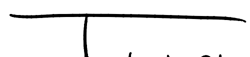


0 shear → no shear
4 pt bending pure bending
constant moment

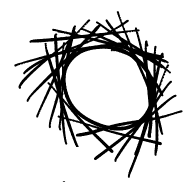




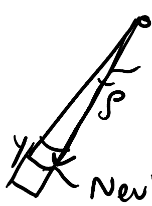
all deformed pieces go through same pt, & are ⊥ to tangent



⊥ & planar x-section



black hole!



Δθ small

Neutral Axis — no stretch or compact

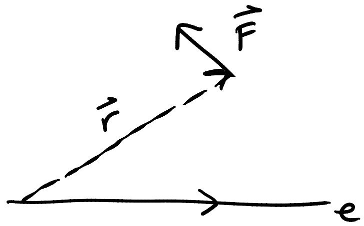
y = distance from neutral axis

$$\overset{\text{strain}}{\varepsilon} = \lim_{\Delta\theta \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\Delta\theta} = \frac{-y}{\rho}$$

Torque, spring const, stiffness const

Wednesday, October 17, 2007
9:56 AM

Torque = moment about an axis

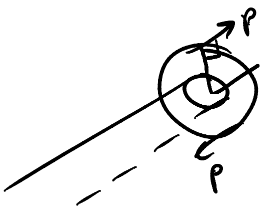


Moment about an axis is a vector.

Define axis w/ unit vector.

$\hat{\imath} \cdot \vec{e} \cdot (\vec{r} \times \vec{F}) = \vec{M}_{axis\ e}(J) = \vec{T}$

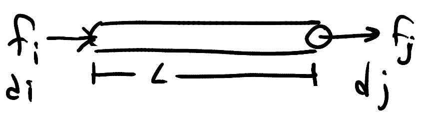
Triple dot product



$2P \cdot r = T$

istanbul,
constantinople now
istanbul,
constantinople now
long time gone
constantinople now
constantinople got the worst...
that's no body's business but the turks!

$\underline{K}$ spring constant, stiffness const.



$f_j = A\sigma$ $\quad\quad \sigma = E\varepsilon$

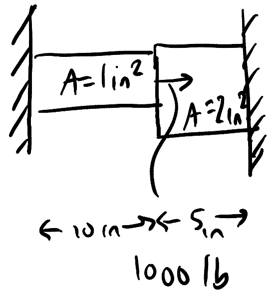
$\delta = d_j - d_i$ $\quad\quad \varepsilon = \dfrac{\delta}{L}$

$$f_i = A\sigma = E\varepsilon A = A\frac{(d_j - d_i)}{L}E$$

$$f_j = -f_i = AE\frac{(-d_j + d_i)}{L}$$

Must be in equilib, which is why they're negative / opp each other.

$$\begin{matrix} f_i \\ f_j \end{matrix} = \frac{AE}{L}\begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}\begin{Bmatrix} d_i \\ d_j \end{Bmatrix}$$



$E_1 = E_2 = 10^7$ psi

Stiffness matrix for each of these.

$\sum F^{int} = \sum F^{ext}$

$$\frac{A_1 E_1}{L_1} = 10^6 \text{ lb/in}$$

$$\frac{A_2 E_2}{L_2} = 4E6 \text{ lb/in}$$

$$k_1 = (10^6)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_2 = 10^6 \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

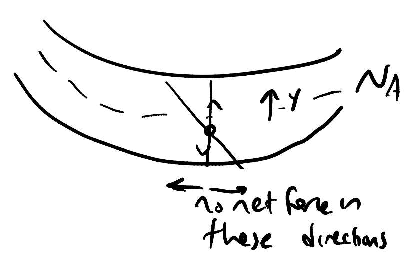
$f_2 = 1000$
f_1 & f_3 unknown

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+4 & -4 \\ 0 & -4 & 4 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

Impose condition $d_1 = 0$, $d_3 = 0$
Reduce to 1 eqn

$d_2 = .2 \times 10^{-3}$ in

Bending. Moments cause bending. ∴ Moment = moment about NA
$\int \sigma_n = 0$ σ_n = normal stresses



Neutral axis = centroid

Strain at any pt

$\varepsilon = -\frac{y}{c}$ y = dist away from NA

C is max dist you can get, .5 from top to bottom

$$\varepsilon = -\frac{y}{c}\varepsilon_{max}$$

$$\sigma = -\frac{y}{c}\sigma_{max}$$

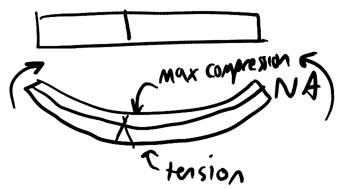
$$\int d\sigma = 0$$

$$\int d\sigma = 0$$

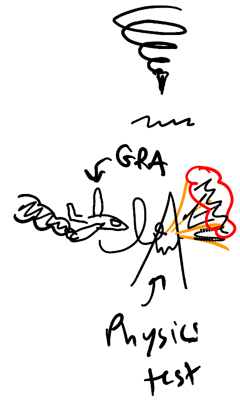
Bending, Moments of Inertia

Friday, October 19, 2007
10:01 AM

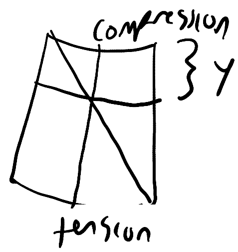
Bending – deformation under a moment



Neutral Axis



$$\varepsilon = -\frac{y}{d}\,\varepsilon_{max}$$



There is no push or pull if only a moment is applied.

C = max dist
Y = dist to NA

$$\sigma = -\frac{y}{c}\sigma_{max}$$

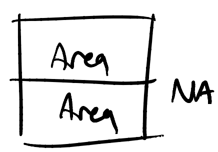
$$0 = \int_A dF = \int_A \sigma\, dA$$

$$= \int_A -\frac{y}{c}\sigma_{max}\, dA$$

$$= -\frac{\sigma_{max}}{C}\int_A y\, dA$$

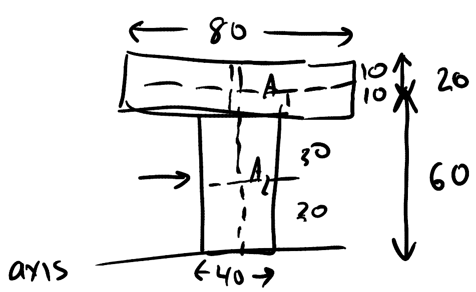
= 0 if we integrate around the centroid. Defines centroid as NA

$\int_A y\,dA$ → 1st Moment of Inertia



Area above = Area below

Centroid of an x-section is the NA.
Location of the centroid goes up + down



Centroid? or NA if bending?

$$\bar{Y} = \frac{\sum A_i \bar{Y}_i}{A}$$

$A_1 = 20 \cdot 80 = 1600$
$A_2 = 40 \cdot 60 = 2400$
4000

$\bar{Y}$

$\int A\, y$

$\int A\, y$

(30)

A_1 70

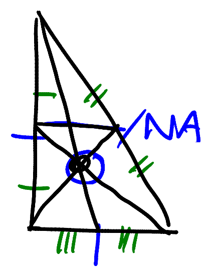
A_2 30

$A_1 y_1 = 112E3$

$A_2 y_2 = 72E3$

$184E3$

$\bar{y} = \dfrac{184E3}{4E3} = 46\text{mm}$



$M = \int_A y\, dF$

$= \int_A y(\sigma\, dA)$

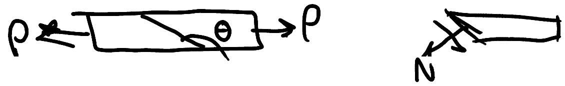
$= \int y\left(\frac{y}{c}\sigma_{max}\right) dA$

$= \dfrac{\sigma_{max}}{c} \underbrace{\int y^2 dA}_{\text{2nd moment of inertia } I}$

$\sigma_{max} = \dfrac{MC}{I}$

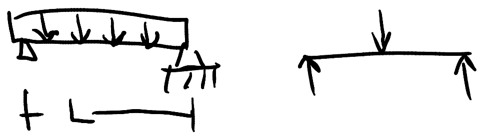
Exam Review

Stresses on inclined sections



$\sigma = E\varepsilon$

Internal forces
- distributed loads

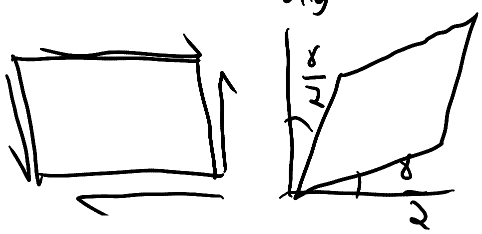


Shear stress / strain

$\tau = \frac{V}{A}$ ← shear force

$\tau = G\gamma$

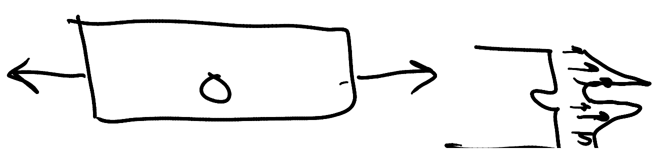
$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$

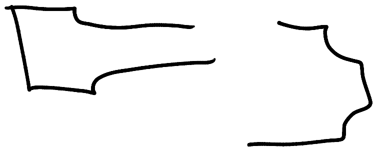


Shear strain is just the ∡

Axially loaded Members
Principle of superposition
Thermal loads $\boxed{\delta_T = \alpha(\Delta T)L}$

Know what stress concentrations look like

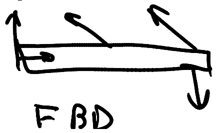




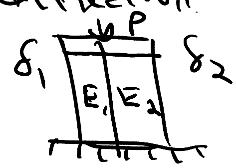
Statically indeterminant Structures

A. Equilib eqns.

B. Compatibility
Some sort of deflection



FBD



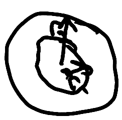
compat statement
$\delta_1 = \delta_2$

C. Constitutive eqns
Hooke's Law

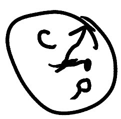
$F = k\delta \quad \sigma = E\varepsilon$

Torsion

$J = \frac{\pi}{2}c^4$ for circular section

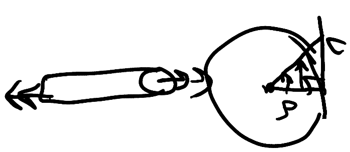


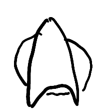
$$\boxed{J = \frac{\pi}{2}(r_2^4 - r_1^4)}$$ (tube)



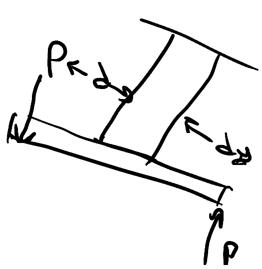
$\tau = \frac{T\rho}{J}$ Shear stress

$\gamma = \frac{\rho\phi}{L_1}$ Shear Strain





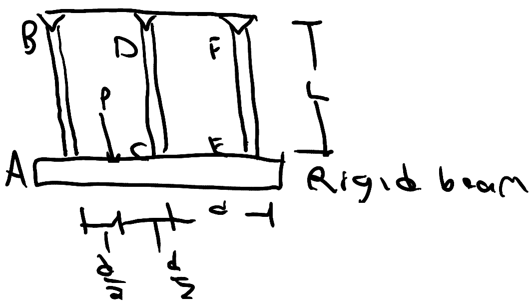
24



$T = 2Pd$
internal torque

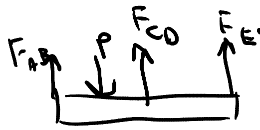
EST: MIAMT

Ex Indeterminant Problem



Eqns for equilib

FBD



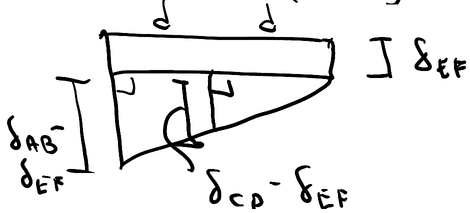
$$\Sigma F_y = F_{AB} + F_{CD} + F_{EF} - P = 0$$

$$\Sigma M_A = -P\left(\tfrac{d}{2}\right) + F_{CD}(d) + F_{EF}(2d) = 0$$

$$-\tfrac{1}{2}P + F_{CD} + 2F_{EF} = 0$$

Compatibility

Columns stretch. Load not in middle so not uniform stretching



Similar Δ's.

$$\frac{\delta_{AB} - \delta_{EF}}{2d} = \frac{\delta_{CD} - \delta_{EF}}{d}$$

$$2(\delta_{CD} - \delta_{EF}) = \delta_{AB} - \delta_{EF}$$

Constitutive eqns

$$F = kx \qquad k = \frac{EA}{L} \Rightarrow F_{AB} = \frac{EA}{L}\delta_{AB}$$

$$F_{CD} = \frac{EA}{l}\delta_{CD}$$

$$F_{EF} = \frac{EA}{L}\delta_{EF}$$

$$\sigma = E\varepsilon$$

$$\frac{P}{A} = E\frac{\delta}{l}$$

BOREDOM!

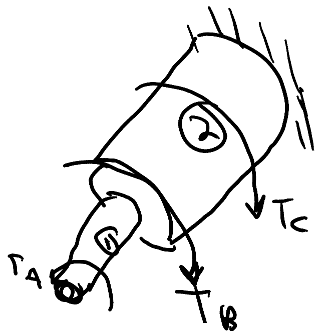
Solve

Sub 4, 5, & 6 into 1.

$$\delta_{AB} + \delta_{CD} + \delta_{EF} = \frac{PL}{EA}$$

$$-\delta_{AB} + 2\delta_{CD} - \delta_{EF} = 0$$

$$3\delta_{CD} = \frac{PL}{EA}$$



$T_A = 5$ kNm

$T_B = 1.5$ kN·m

$T_C = 12$ kN·m

Max shear stress?

$d_{①} = 80$mm

$d_{②} = 120$mm

$T_{1.5} \rightarrow T_{AB}$

$T_{AB} = 5$

$5 \quad 1.5 \rightarrow T_{BC}$

$$\Sigma T_x = 0$$

$$-5 + 1.5 + T_{BC} = 0$$

$$T_{BC} = 3.5 \text{ kN·m}$$

$5 \quad 1.5 \quad 12 \rightarrow T_{CD}$

$$\Sigma T_x = 0$$

$$-5 + 1.5 + 12 + T_{CD} = 0$$

$$\Rightarrow T_{CD} = -8.5 \text{ kNm}$$

$$\tau = \frac{Tc}{J} \qquad J_{①} = \frac{\pi}{2} c^4 = \frac{\pi}{2}(.04)^4$$

$$J_{②} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{2}(.06)^4$$

Tips

FBD!

Check units

Indeterminant problems
- FBD has only rigid member + forces

Practice Problems

1-86

1-117 Pt D only

3-29

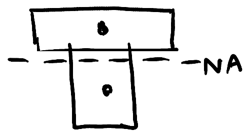
4-61

5-61

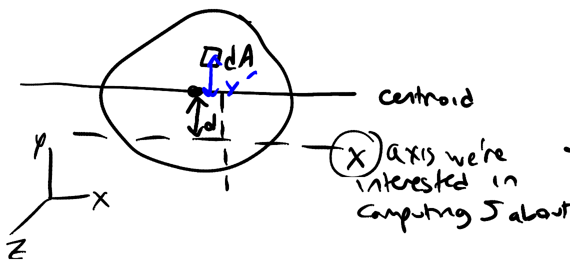
Use FBD's

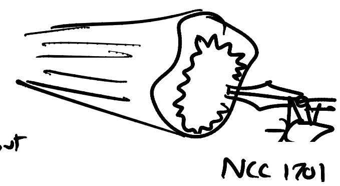
Bending is something caused by a moment
Pure bending results from only a moment. No shear, no tensile/compressive stress

Bends into an arc



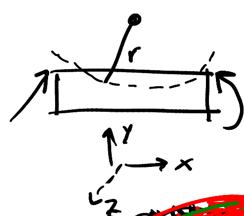
2nd Moment of Inertia about the centroid of a shape is needed for these problems.





Z axis coming out of dot.

When we want the J of a complicated shape we want the J of the NA



$$I_x = \int y^2 dA = \int_A (y'+d)^2 dA$$

$$I_x = \underbrace{\int_A y'^2 dA}_{\bar{I}_{x'}} + \underbrace{\int_A 2y'd\,dA}_{\int_A y'dA \;=\; 0} + \underbrace{\int_A d^2 dA}_{d^2A}$$

d is a const.

b/c centroid has area above = area below.

$$I_x = \bar{I}_{x'} + d^2A$$

know this, know this

look up in book. 2nd moment of inertia around x'.

I_x = defined here

CRO-MAGNON ~~MAN~~ MATT

2nd moment of inertia above NA ≠ 2nd moment of inertia below NA

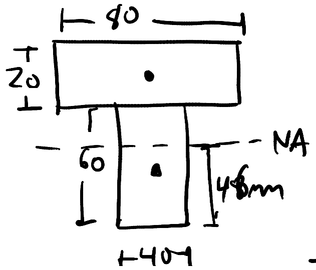
1st moment of inertia above NA = 1st moment of inertia below NA

2nd moment of inertia above NA = applied moment that causes bending.

If 2nd above was = 2nd below, it wouldn't bend →

this is what causes bending

this is what causes bending



Calculate I.
Maximum stress on beam is the farthest pt from the centroid.

$$\bar{y} = \frac{\Sigma A_i y_i}{A_T} = \frac{1}{400}(1600)(70) + 2400(30) = 46\text{mm}$$

Cheat sheet

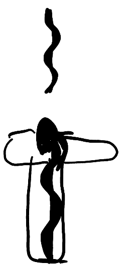
$$\boxed{I_i = \frac{1}{12}bh^3}$$

For each of the 2 separately

$$I_{x'} = \Sigma(\bar{I}_i + Ad^2) \qquad \text{Parallel Axis theorem}$$

$$I_{x'} = \bar{I}_A + A_A d_A^2 + \bar{I}_B + A_B d_B^2$$

$$= \underset{\bar{I}_A}{5.33E4} + 1600(70-46)^2 + \underset{\bar{I}_B}{(72E4)} + (2400)(46-30)^2$$



$$= 230.9E4 \text{ mm}^4$$

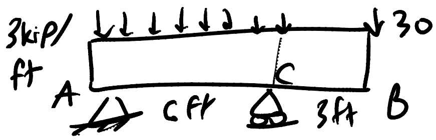
$$\sigma_{max} = \frac{MC}{I}$$

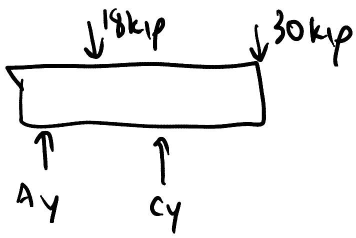
$$= 398 \text{ MPa}$$

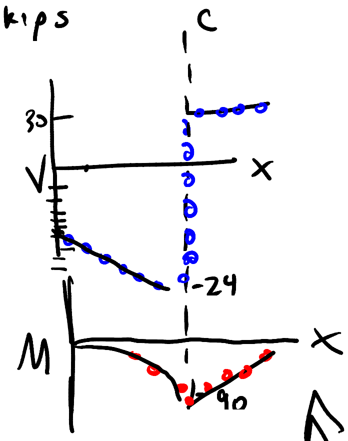
Sample Prob Forces Moments

Friday, October 26, 2007

10:00 AM







$\Sigma F_y = A_y + C_y - 18 - 30 = 0$

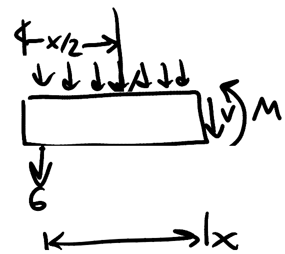
$\Sigma M_A = -18(3) + C_y(6) - 30(9) = 0$

$C_y = 54$

$A_y + 54 - 18 - 30 = 0$

$A_y = -6$

From A to C

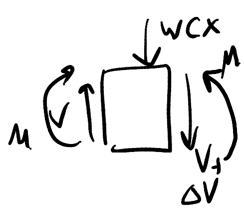


$\Sigma F_y = -6 - 3x - V = 0$

$V = -6 - 3x$

$\Sigma M = 0 = 6x + 3x\left(\frac{x}{2}\right) + M$

$M = -1.5x^2 - 6x$



Wherever there's a concentrated load, look for a discontinuity = mag. of concentrated load

$V - w\Delta x - V - \Delta V = 0$

$\frac{dV}{dx} = -w(x)$

$\Sigma M = 0$

$-V\Delta x + w\Delta x\,\Delta x - M + M + \Delta M = 0$

$$-V\Delta x + w\Delta x \frac{\Delta x}{2} - M + M + \Delta M = 0$$

$$\Delta M = V\Delta x - \frac{w\Delta x^2}{2} \qquad \text{As } x \to 0\text{, solve for } M$$

$$\boxed{\frac{dM}{dx} = V(x)}$$

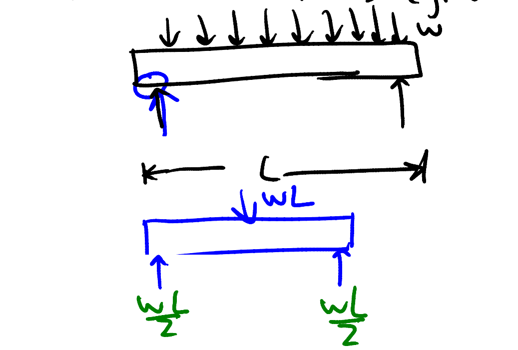
$$\boxed{\frac{dV}{dx} = -w(x)}$$

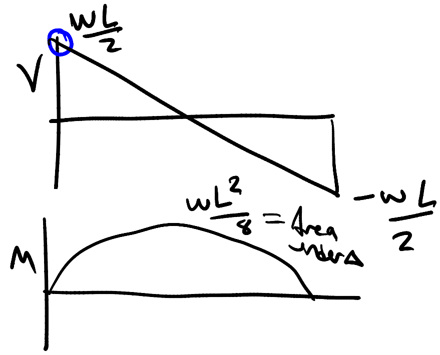
Shear & Moment Diagrams

Tuesday, October 30, 2007

10:10 AM

Shear & Moment Diagrams

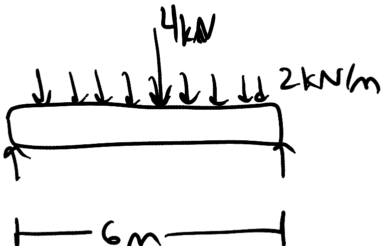


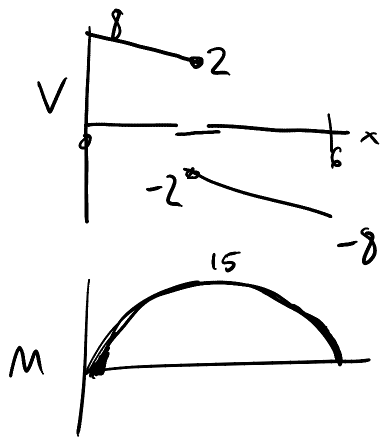


$$\frac{dV}{dx} = -w$$

$$\frac{dM}{dx} = V$$

$$\frac{d^2M}{dx^2} = \frac{dV}{dx} = -w$$

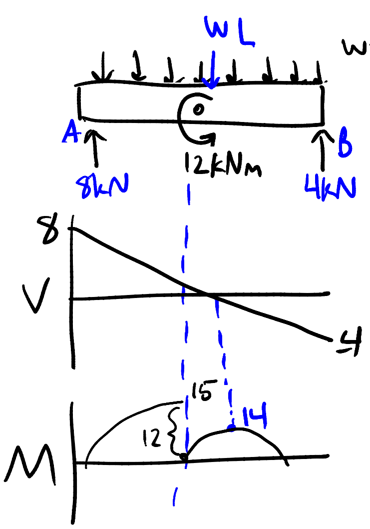


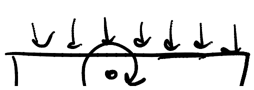


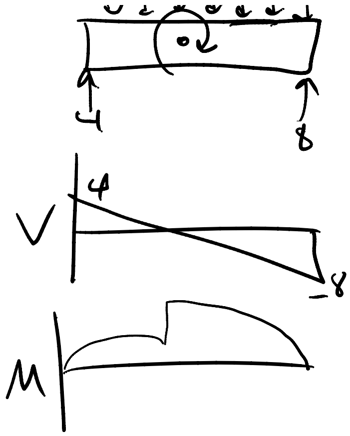
w=2 therefore

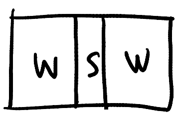
$$\Sigma M_A = 0 = 6w(3) + 12 - 6B_y = 0$$

$$\Sigma F_y = A_y + B_y - 6w = 0$$





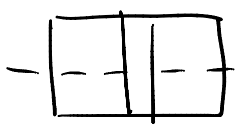




Bend about z axis
coming out of screen.

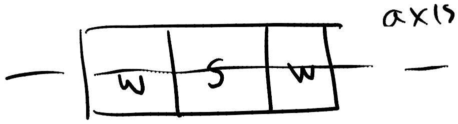
Steel's Young's modulus E = 200 GPa
Wood's Young's modulus E = 12.5 GPa

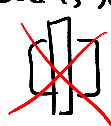
Neutral axis drawn



N.A. = Neutral surface

Make beam out of 1 material, wood.
Make steel bigger, wider, stretched along neutral axis so wood is same but steel is stretched.

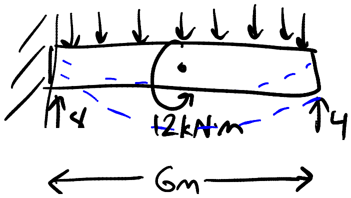




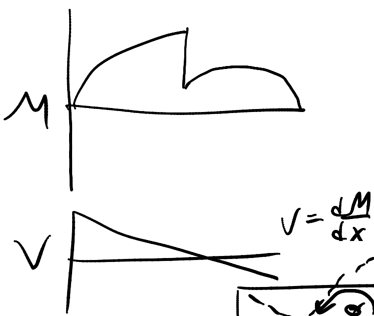
This is bad.

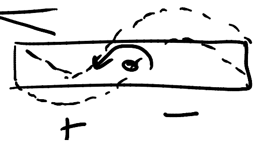
Distributed Load, Sample Prob

Wednesday, October 31, 2007
9:57 AM



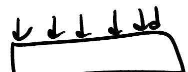
12 kN·m is CCW, (+), so why do we go down from 15 to 3 kN·m on the M diagram?



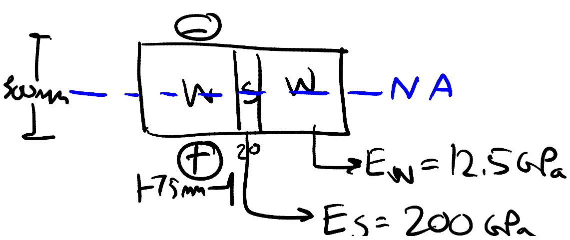


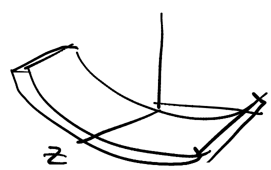
Bends in an S.

+ to − means drop, since diagram is superimposed on object.
− to + means climb.

Superimpose this on the 
And it decreases the bending on the right side & increases it on the left side

Positive is concave up





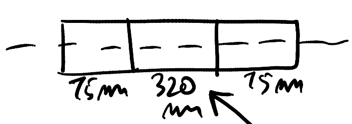
bends in z axis

Concave up is ⊕ tension
Concave down is ⊖ Compression

$$\sigma_{max} = \frac{MC}{I}$$

M is moment
C is max distance from NS to the edge

$$I = \frac{1}{12} b h^3$$



Make stronger thing bigger or weaker thing smaller.

$$\boxed{\left(\frac{E_S}{E_W} = 16\right) 20 = 320} = n$$

We converted steel to wood by making steel bigger.

Now we have an equivalent problem

$I = 1.0575E3\ m^4$ $\qquad$ $M_{given} = 10\ kN \cdot m$

$$\sigma_{max} = \frac{(10kN\cdot m)(.15\,m)}{1.0575E3\,m^4} = 1.418\,MPa$$

in wood

To get σ_{max} in steel, shrink steel down

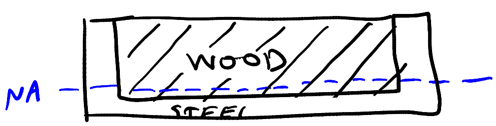
$$\sigma_{max} = (\sigma_{max}\ \text{wood})\,n$$
$$= 22.7\,MPa$$

Steel takes greater amt of stress. It's stronger & stiffer.

Wood is less stiff so you need more of it to carry the same loads. That's why we needed a bigger piece of wood to equal the piece of steel.

Steel takes greater amt of load not b/c it's strong but b/c it's stiff. B/c it's strong, it's ok that it takes more of the load.

Glass is very strong, stronger than steel, but it's easily scratched & stress concentrations makes it very easy to break.



$M = 850$ lb·ft

Wood = 15 in × 3.5 in

Steel = .5 in thick

$E_W = 1600$ ksi

$E_S = 29E3$ ksi

Bending along same axis as before

Shrink the wood

Find neutral surface.

$$\bar{y} = \frac{\sum A_i \bar{y}_i}{A_{total}}$$

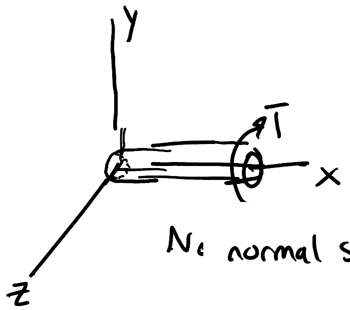
Parallel axis theorem.

Normal stress tends to stretch or shrink things.

Formula for Shear stress when only torque is applied:

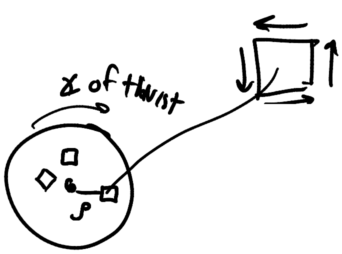
$$\boxed{\tau = \frac{T\rho}{J}}$$

1st polar moment of inertia



No normal stress, only shear.

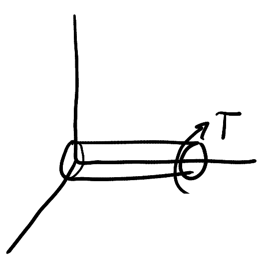
τ is in the zy plane. $\perp$ to axis of rotation.
Max shear stress is at the surface.

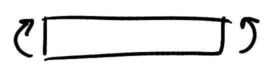


Shear stress

Bending

The greater the bending the greater the tension & compression.





Concave up, (+)

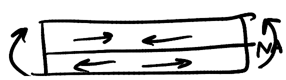
Bending results in normal stress || to NA

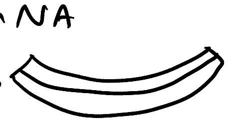
$$\boxed{\tau = \frac{T\rho}{J}}$$

$$\boxed{\sigma = \frac{MC}{I}}$$

Max dist from NA

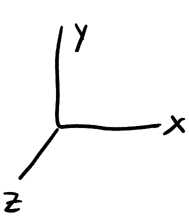
2nd moment of inertia





but then there's shear. If you bend a ~~pile~~ tablet of paper the top papers are in compression & the bottom in tension. They're also rubbing up against each other.

Shear is in xy plane.



.5 in thick
15 in

z'

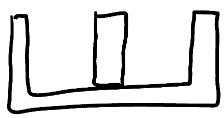
$E_W = 1600$ ksi
$E_S = 29E3$ ksi

$M = 850$ ft lb

Max normal stress?
What's the normal stress at some position?

We need to know I, therefore find NA = NS = Centroid

$$\bar{Y} = \frac{\Sigma(A_i \bar{Y}_i)}{\Sigma A_i} = \frac{(.5)(16)(.25) + 2(3.5)(.5)(2.25) + (.8276)(30)(2.25)}{(.5)(16) + 2(3.5)(.5) + (.8776)(3.5)}$$



$= 1.1386$ in above bottom

$$I = \Sigma \bar{I} + Ad^2$$

Parallel axis theorem where d = dist from NA to the centroid of the individual members.

$I = 20.914$ whole section $\quad 4 - 1.1386 = c$

$4 - 1.1386 > 1.1386$

$$\sigma = \frac{MC}{I}$$

$$= \frac{850\left(\frac{12\,in}{ft}\right)(\text{greater stress at greater of 2 distances away.})}{20.914}$$

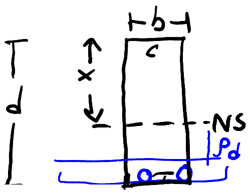
$= 1.4$ ksi Max stress in steel.

Stress in wood will be smaller.

$$(1.4\text{ ksi Max stress in steel})\left(\frac{1600\text{ ksi}}{29E3\text{ ksi}}\right) = \_\_\_\_$$

E in wood

~~Ste~~ E in steel



Comes in & out of board
Pretend the bars are squares.
steel

Reinforced concrete
No strength in tension.
Don't put reinforcing bars in the middle b/c there's nothing there.

Tension & compression modulus are generally the same.

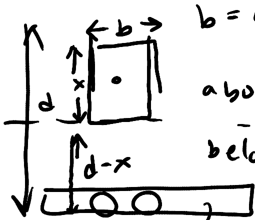
Modulus is no longer constant. We have 3 materials

$\boxed{\text{1st moment of inertia about the NA} = 0}$

$$Q_x = \Sigma A_i \bar{y}_i = 0$$
$$= (bx)\frac{x}{2} + nA_s(-(d-x))$$

n is ratio of steel modulus to concrete modulus

modulus,



$b = 4$ in.

above N.S. = (+)

— — — nS

below N.S. = (−)

$d = 8$ in

transformed rebars

$$n = \frac{E_{steel}}{E_{concrete}}$$

This is concrete w/ rebars. But there's a gap b/c the concrete below the neutral surface can't hold tension.

The neutral axis /centroid is where the 1st moment of inertia is zero. $Q_{ns} = 0 = \Sigma A_i y_i = bx\left(\frac{x}{2}\right) + nA_s(-(d-x))$

$$\frac{1}{2}bx^2 + nA_s x - nA_s d = 0$$

$M = 40$ kip·in

$E_s = 30E6$ psi

$E_c = 3.75E6$ psi

$n > 1$

$\frac{30E6}{3.75E6} = 8$

$A_s = \frac{\pi}{4} = .7854 \text{ in}^2$

$nA_s = 6.283 \text{ in}^2$ $\qquad X = 3.683$

Max stress of concrete in compression is at top.

And max stress of steel is at bottom.

n is always a ratio

We are computing normal stress, tension or compression. W/ normal stress, we have bending.

$$\sigma_{top} = \frac{Mc}{I} = \frac{(40 \text{ kip in})(x)}{\frac{1}{12}bh^3} = \frac{(40)(3.683)}{\left[\frac{1}{12}(4)(3.683)^3 + 4(3.683)\left(\frac{3.683}{2}\right)^2\right]}$$

Parallel axis theorem

I_1

$$\sigma_{max\ bottom} = \frac{40(d-x)}{\frac{1}{2}(6.283)(1)^3 + 6.283(4.317)^2}$$

I_2

$$I_{total} = I_1 + I_2$$

$$I_{total} = I_1 + I_2$$

In concrete $\quad \sigma_{max} = \dfrac{-40E3(3.683)}{I_1 + I_2} = -800\,psi$

Neg b/c compression at top.

In steel $\quad \sigma_{max} = n\left[\dfrac{(40E3)(8-3.683)}{I_1 + I_2}\right] = 7500$ or 7.5 ksi

$n = -8$

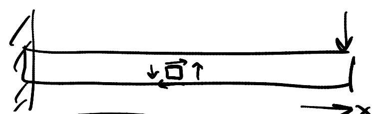
Bending: $\sigma = -\dfrac{Mc}{I} = \boxed{-\dfrac{My}{I}}$

$\tau = \dfrac{Tc}{J} = \boxed{\dfrac{T\rho}{J}}$

$\sigma_N = \dfrac{P}{A}$

Patrick Brady O'Malley's Essay on Sacrifice

We all have to make sacrifices. Sometimes we give up TV shows to do chores, not b/c we want to but b/c we need to. Aedan and I have sacrificed school because we need to. We still go to school at home, but they won't let us go to Brookview because of my Mom's job. People are scared that we'll make criminals come to school. Mr. Schumacher is our teacher and he tries to make school fun.

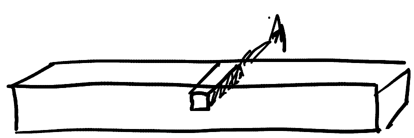


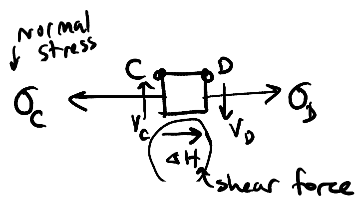
$$V = \frac{dM}{dx}$$

Applied forces we can change that results in shear? Distributed load, 4 pt bending.

If we have a const. moment, there is no shear. Pure bending has const moment.

Shear is ∥ to surface in x direction





Shear on a free surface = 0

$$\Sigma F_x = 0 = \int_A \sigma_D\,dA - \int_A \sigma_C\,dA + \Delta H = 0$$

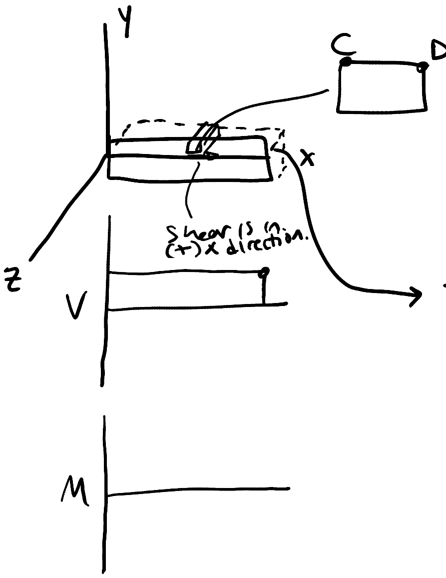
The diff between $V_C + V_D = \Delta H$, b/c of physics of shear & moment diagrams.

$$\Delta H = -\int (\sigma_D - \sigma_C)\,dA$$
$$\Delta H = \int \left(\frac{M_D y}{} - \frac{M_C y}{}\right) dA$$

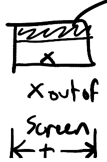
$$= \frac{M_D - M_C}{I} \underbrace{\int y \, dA}_{Q}$$

Shear Flow, Shear stress

Tuesday, November 06, 2007
11:00 AM



$$\frac{\text{Shear force}}{\text{Area}} = \text{stress}$$



X-section

ΔH = shear force inside beam $= \dfrac{M_D - M_C}{I} \underbrace{\int y\,dA}_{Q}$

$$\frac{\Delta H}{\Delta x} = \frac{\Delta M Q}{\Delta x I} = \frac{VQ}{I}$$

V ← shear force outside beam.

$\Delta H \to 0$,

$$q = \frac{VQ}{I}$$

q ← shear flow; Q ← 1st moment of area; I ← 2nd moment of inertia

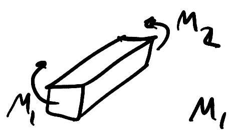
Shear Flow - shear force acting over whole length

Shear Stress - Shear force per unit area

$$\frac{q}{t} = \tau$$

$$\tau = \frac{VQ}{It}$$

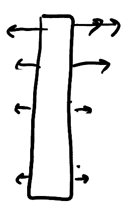
Avg shear stress



$M_1 \neq M_2$, so causes a shear force

Shear flow units: $\dfrac{F}{A} t$ ← t = thickness.

$$\frac{F}{l} = \frac{N}{m}$$

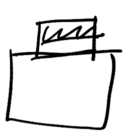


$$\sigma = \frac{My}{I}$$

y = dist from NA

By statics, $\sigma = 0$ b/c $\Delta M = 0$. $M_1 = M_2$

→ Shear Force → $\Sigma F_x = 0$

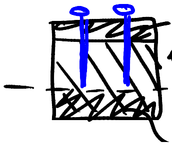


Shear force is coming out of the board

$$\Delta H = \frac{M_D - M_C}{I}$$

Q for this area?

Q is the negative of the stuff on the top.



$Q_{top} = -Q_{everything\ else}$

NA

neg of top, so cancel out.

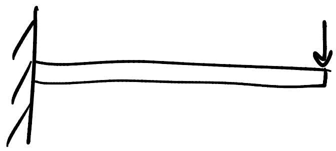
Each nail takes half of the shear flow

A changing moment means that there's a shear.

Most common kind of failure is fatigue - cyclic loading

4 Point Bending

Wednesday, November 07, 2007
11:03 AM



$$\frac{dM}{dx} = V$$

4 pt bending is a very good way of applying pure bending force.

pure b/c moment doesn't change, V=0

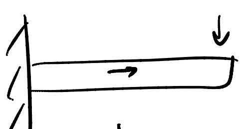
This is important b/c we can compute the value of the strength & modulus during pure bending.

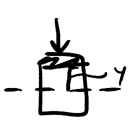
$$\sigma_{max} = \frac{Mc}{I}$$

If we know the load we can compute the moment & measure the strength. This is very close to this value!

$$\sigma_M = \frac{P}{A}$$

As a matter of practicality it's easier to bend the sample than pull it apart.



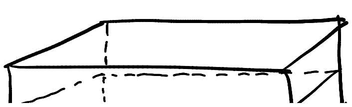


applied shear force

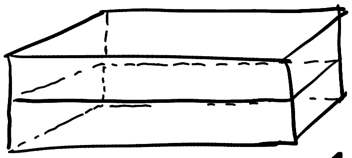
$$q = \frac{VQ}{I}$$

← Moment of area about NA $= Q = A\bar{y}$

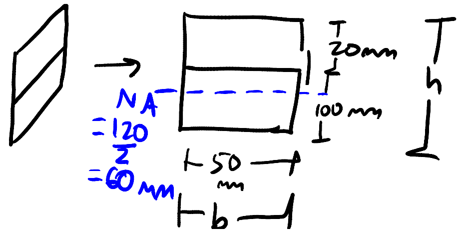
2nd moment of inertia $\frac{1}{12}bh^3$



$V = 800N$



$V = 800N$



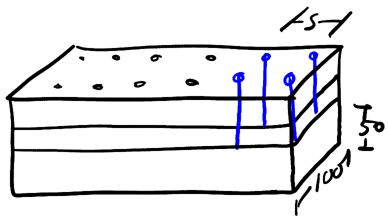
$\frac{q}{t} = 0$

$t = b$ here

$\frac{q}{t} = 0$

$$\frac{\overbrace{(800N)}^{V}(\overbrace{(.02)(.05)}^{A}\overbrace{(.05)}^{\bar{y}})}{\underbrace{\frac{1}{2}(.05)(.12)^3}_{I}\underbrace{(.05)}_{b}} = 111.1\text{ kPa}$$

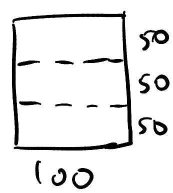
$\frac{VQ}{It} = 0$



$V = 1500N$

Allowable shear stress in each nail is 400N

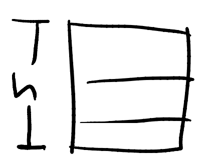
$V_{allowed\ in\ nail} = 400N$



Biggers, greater stress.

$$\frac{\Delta H}{\Delta x} = \frac{VQ}{I} = q$$

Shear flow - how much shear do we get in x direction as a result of the shear.

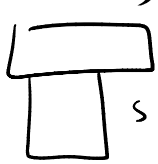


$Q = .050(.100 \cdot .05)$

$I = \frac{1}{2}(.100)(.15)^3$

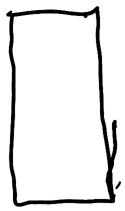
If 2 diff materials,

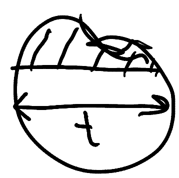
$$s = \frac{I\,(V)_{nails}}{V_{max}(Q)} = \frac{I\,(400N)}{(1500N)(Q)}$$



shank stronger

Affects $\bar{y}$.

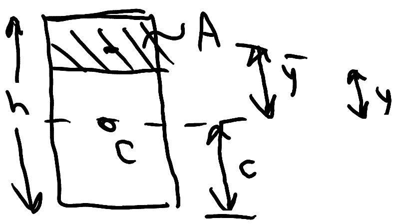




$$q = \frac{VQ}{I}$$

shear flow being computed anywhere along the base of the cross section.

$t = b$ for a rectangle



$$Q = A\bar{y} = b(c-y)\left(y + \tfrac{1}{2}(c-y)\right)$$

$$= b(c-y)\tfrac{1}{2}(c+y)$$

$$= \frac{b}{2}(c^2 - y^2)$$

$t = b$

$NA = \frac{h}{2}$, $h = 2c$

$$I = \frac{1}{12}bh^3 = \frac{2}{3}bc^3$$

$$q = \frac{V\left(\frac{b}{2}(c^2 - y^2)\right)}{I}$$

$$qt = \tau = \frac{QV}{It} = \frac{V\left(\frac{b}{2}(c^2 - y^2)\right)}{\left(\frac{2}{3}bc^3\right)b} = \frac{3}{4}\left(\frac{c^2 - y^2}{bc^3}\right)V$$

$A' = b \cdot h \leftarrow A' = 2bc$ (area of cross section)

different area

$$\tau_{xy} = \frac{3}{2}\frac{V}{A'}\left(1 - \frac{y^2}{c^2}\right)$$

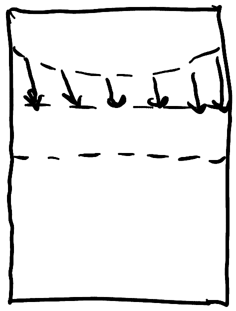
when $y = c$, τ at min
when $y = 0$, τ at max

$$\boxed{\tau_{max} = \frac{3}{2}\frac{V}{A'}}$$

$$\sigma_{max} = \frac{Mc}{I}$$

$\sigma = \frac{My}{I}$, c is max value

$\tau_{Avg} = \frac{VQ}{It}$ It's zero at top, max in middle, & zero at bottom



greater at edges than in the middle.

$b = \frac{h}{4}$ given

$$\frac{\tau_{max}}{\tau_{avg}} = 1.008$$

If $b = h$, $\frac{\tau_{max}}{\tau_{avg}} = 1.126$

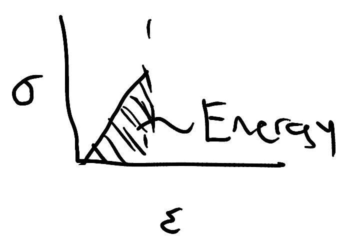
Better for long thin things than short fat things.

There is no shear on a free surface

Free surface - atoms right on the surface of a plane.

Surface $\perp$ to

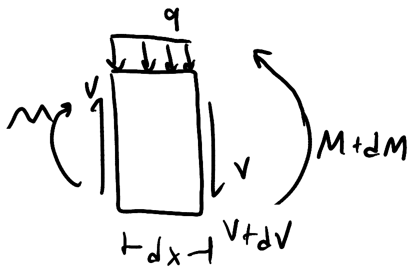
Lalala La La



Force · distance = E

Example Prob

Monday, November 12, 2007
11:02 AM



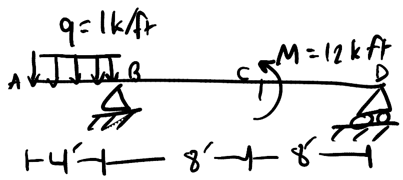
$$\Sigma F_y = 0 = V - q\,dx - (V + dV) = 0$$

$$\frac{dV}{dx} = -q$$

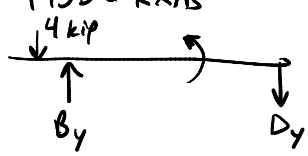
$$\Sigma M = 0 = -M - q\,dx\left(\frac{dx}{2}\right) - (V + dV)dx + M + dM = 0$$

$$\frac{dM}{dx} = V$$

Example



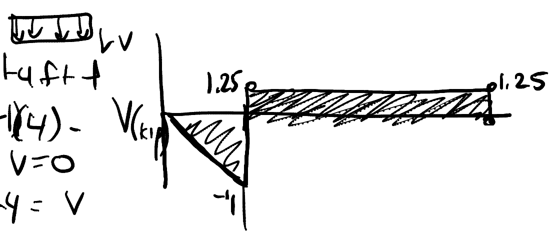
1) FBD & Rxns



$\Sigma F_y = 0$ etc

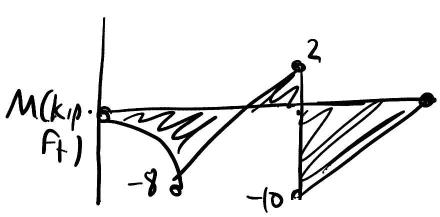
$B_y = 5.25$ kip ↑

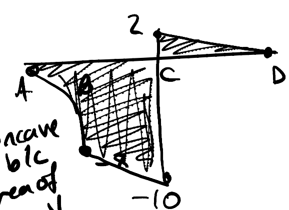
$D_y = 1.25$ kip ↓



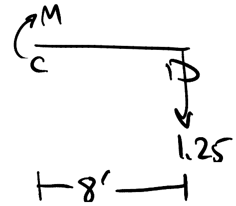
$(-1)(4) - V = 0$

$-4 = V$





concave ↓ b/c area of Δ on V is neg

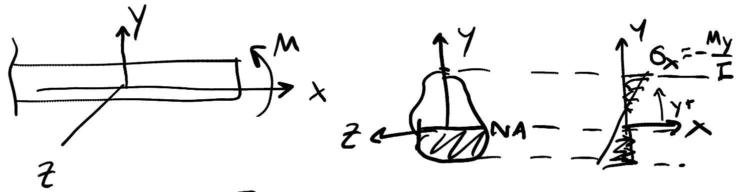


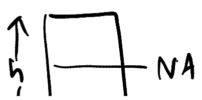
$\Sigma M_c = 0$

$-M_c - 1.25(8) = 0$

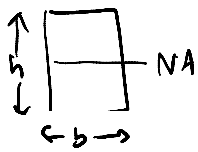
-10 kip ft $= M_c$

Normal stresses





$I_{NA} = \frac{bh^3}{12}$



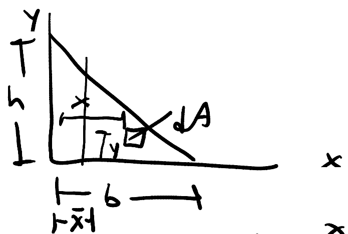
$I_{NA} = \frac{bh^3}{12}$

$Q_x = \int y\,dA$

$Q_y = \int x\,dA$

$\bar{x} = \frac{Q_y}{A_t}$

$\bar{y} = \frac{Q_x}{A_t}$



$y = h(1 - \frac{x}{b})$

$Q_x = \int y\,dA$

$Q_x = \int_0^b \int_0^{h(1-\frac{x}{b})} y\,dy\,dx = \int_0^b \frac{y^2}{2}\Big]_0^{h(1-\frac{x}{b})} dx$

$= \int_0^b \frac{h^2(1-\frac{x}{b})^2}{2}\,dx$

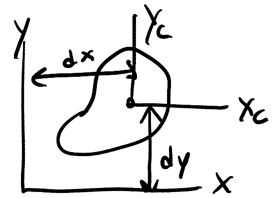
$= \frac{bh^2}{6}$

$\bar{y} = \frac{Q_x}{A} = \frac{bh^2}{\frac{1}{2}bh} = \frac{h}{3}$

$\bar{X} = \frac{\sum_{i=1}^{n} \bar{x}_i A_i}{\sum_{i=1}^{n} A}$ $\qquad \bar{y} = \frac{\sum_{i=1}^{n} \bar{y}_i A_i}{\sum_{i=1}^{n} A}$

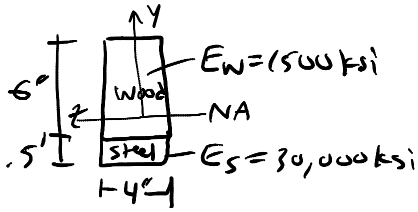
Moment of inertia

$I = \int y^2\,dA$

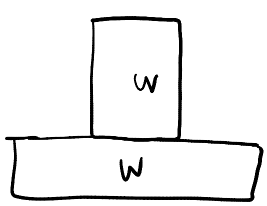


$I_x = I_{x\perp} + (A d_y)^2$

Transformed section method

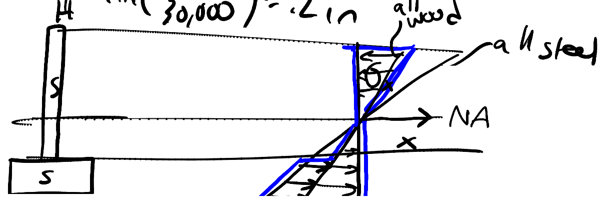


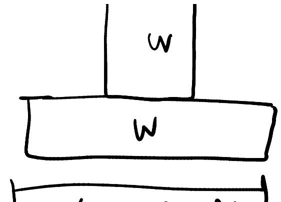
Want to get NA



OR

$4in\left(\frac{1500}{30,000}\right) = .2\,in$

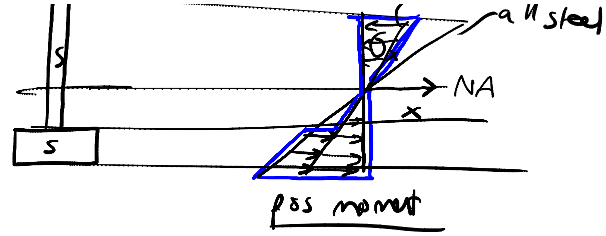




$4\,in^2\left(\frac{30{,}000}{1500}\right)$

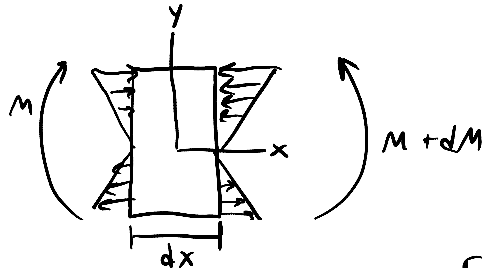
$= 8.0\,in$

OR



Shear

$$\tau = \frac{VQ}{It} = \frac{VQ}{Ib}$$



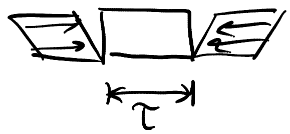
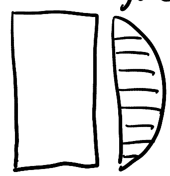
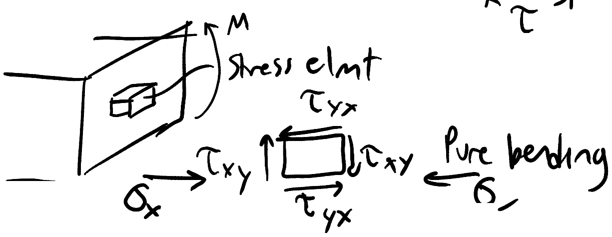
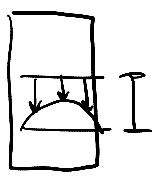


Figure 7-5

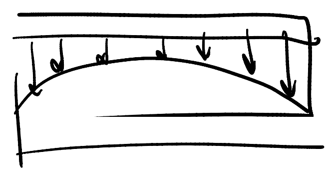






$$\tau = \frac{VQ}{Ib}$$

cross-section



works for a narrow beam.

S Beam (I beam)

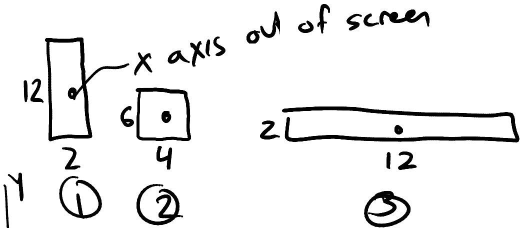
Tuesday, November 13, 2007
11:32 AM

$$\sigma_{max} = \frac{Mc}{I} \qquad S = \frac{I}{c} \qquad S = \text{section modulus}$$

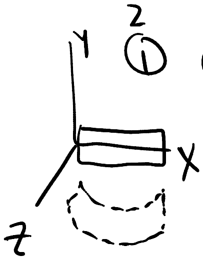
$$= \frac{M}{S}$$

S for a rectangular x-section $= \dfrac{\frac{1}{12}bh^3}{\left(\frac{h}{2}\right)} = \frac{1}{6}bh^2 = \frac{1}{6}Ah$

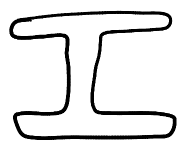
large # to keep σ_{max} low



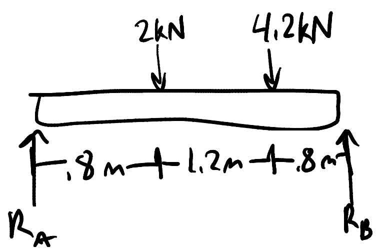
$A = 24$



So S_1 is biggest for bending around z axis.



I beam has a very high S. Also called an S beam for this reason.

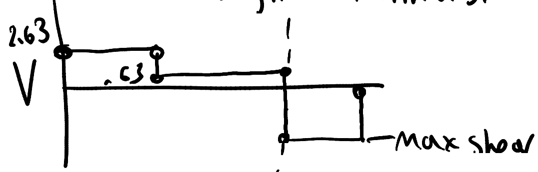


Eccentrically loaded

$\sigma_{allow} = 14$ MPa
$\tau_{allow} = 725$ kPa

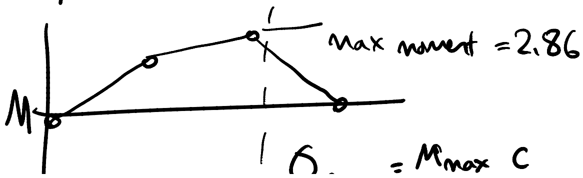
width into board = 40mm

height? Normal stress?



$\Sigma F_y = 0$ etc
$\Sigma M_A = 0$

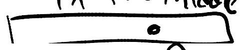
$R_A = 2.63$ kN
$R_B = 3.57$ kN



$\sigma_{.} = \frac{M_{max}\, c}{}$ $\qquad M_{max}$

$$\sigma_{max} = \frac{M_{max}\, c}{I} = \frac{M_{max}}{S}$$

max shear stress is here. in the middle



$$S = \frac{M_{max}}{\sigma_{allow}} = \frac{2.86E3}{14E6} = 2.043E{-4}\ m^3$$

$$S = \frac{1}{6} b h^2 = \frac{1}{6}(40mm)(h^2) = 2.043E{-4}\ m^3$$

$$h = 175\ mm$$

$$\tau = \frac{3}{2}\frac{V_{max}}{A = bh}$$

$$\tau_{allow} = 725\,kPa = \frac{3}{2}\frac{V_{max}}{bh}$$

V_{max} ← from diagram

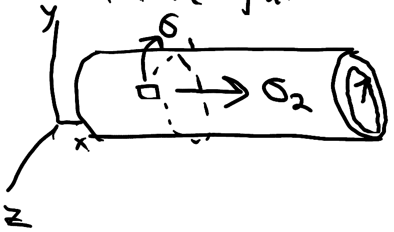
h ← solve 4 this

b ← know this

Thin-walled vessel

Friday, November 16, 2007
11:03 AM

Thin-walled vessel is a container that holds a fluid/gas.



t = thickness

$$\frac{r}{t} \geq 10$$

These analyses fall apart when the wall is thick compared to the radius

Pressurizing interior generates normal & shear stress

Pure bending, normal stress

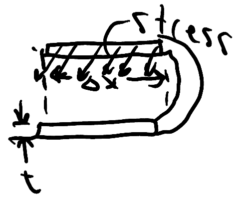
$$\sigma = \frac{My}{I}$$

Can apply twist, shear stress.

Normal stress only for this problem

σ_1 = hoop

σ_2 = longitudinal



$$\Sigma F_H = 0 = 2\sigma_1 \Delta x t$$

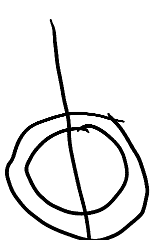
Force = stress(x sectional area)

$$2\sigma_1 \Delta x t = P 2 r \Delta x$$

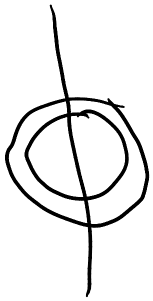
pressure

area of $\frac{1}{2}$ plane coming out

$$\boxed{\sigma_1 = \frac{Pr}{t}}$$ hoop stress



stresses coming out of page = longitudinal



stresses coming out of page = longitudinal

$$\Sigma F_x = 0 = \sigma_2 (2\pi r t = P(\pi r^2)$$

$$\boxed{\sigma_2 = \frac{Pr}{2t}}$$ longitudinal

σ, 2x σ_2

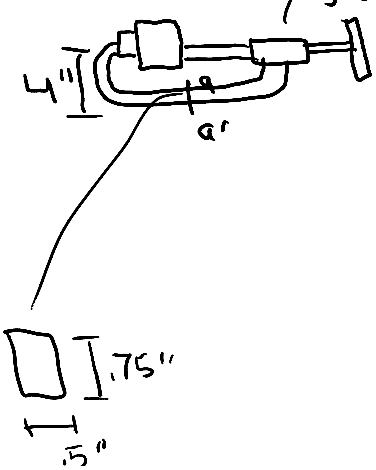
Spherical pressure vessel

This is the same σ as the longitudinal stress in <u>any</u> direction, & they're always equal.

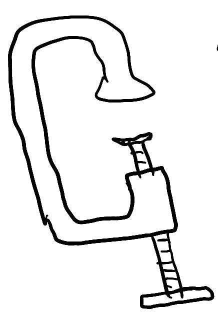
$$\sigma_2 = \frac{Pr}{2t}$$

A sphere minimizes the stress

500 lb force



This is Proffessor Jennings' version of a C-clamp



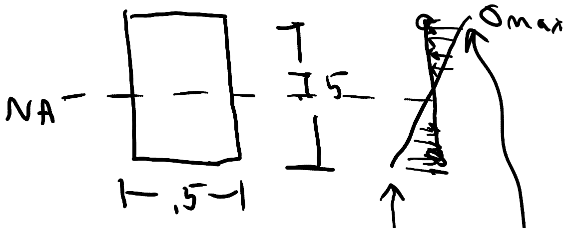
M = -2000 lb-in

No torsion

Map stresses

Axial load; axial stress $\sigma_{axial} = \frac{500}{A} = \frac{500}{.375} = 1.33$ ksi

It isn't a pure moment but we can break it down into pure moment + axial moment + torsion moment = total



$$\sigma_{max} = \frac{Mc}{I} = \frac{M\left(\frac{.75}{2}\right)}{\frac{1}{12} b h^3}$$

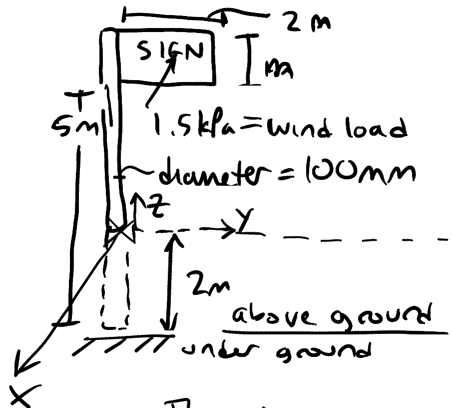
$$= \frac{M\left(\frac{.75}{2}\right)}{\frac{1}{12}(.5)(.75)^3}$$

$M = 2000 \text{ lb in}$

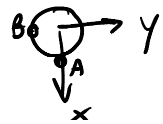
$\sigma_{max} = 42.7 \text{ ksi}$

$\sigma_{max\,a} = 44 \text{ ksi}$ $\quad 42.7 + 1.33$

$\sigma_{max\,a'} = -41.37$ $\quad 1.33 - 42.7$



Wind is blowing down x axis in neg x direction.

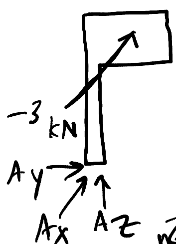


There is a rxn under ground but we want to know the forces at the surface.

1.5 kPa is distributed over the SIGN.

Replace w/ single load.

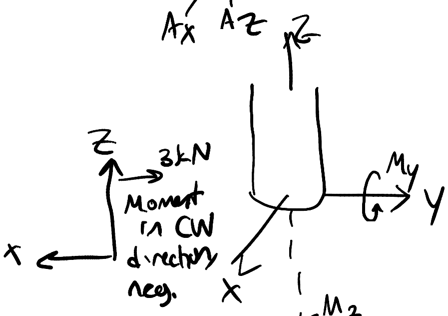
$1.5\,kPa\,(2)(1) = \underline{-3kN}$



$\Sigma F_x = 0 = A_x - 3kN = 0$

$A_x = 3kN$

$\Sigma F_y = 0 = A_y$

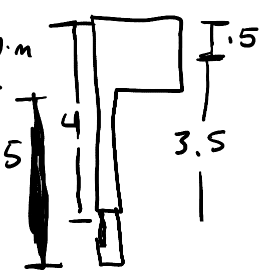


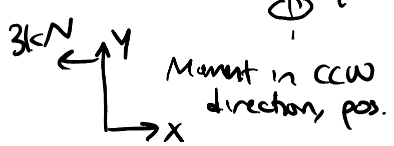
$P_x = -3kN$

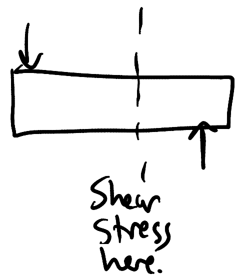
kN·m

$M_y = 3.5(3) = -10.5$

$M_z = (3)(1m) = 3$







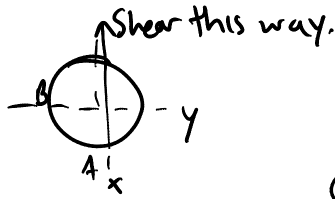
$I = \frac{1}{4}\pi r^4$

$= 4.91E{-6}\,m^4$

$\tau = \frac{VQ}{It}$ Shear force

$\sigma = \frac{M_y c}{I}$ bending

$\tau = \frac{Tc}{J}$ torque / torsion



A

②

J Torque / Torsion

Ⓐ $4'$ x

Ⓑ

$\tau_A = 0$ b/c $V = 0$ on edge A.

$$\tau = \frac{VQ}{It}$$

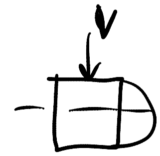
0

$(.1)$

$\tau_B = .51$ MPa

$$Q = \frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)$$

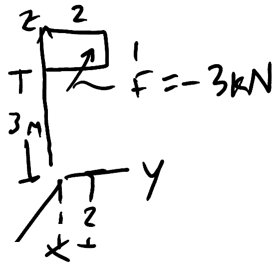
given in book

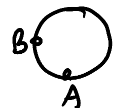


shear stress max in middle

Circle

Tuesday, November 20, 2007
11:00 AM



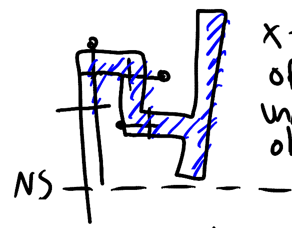


F provides a shear force, torque, + bending moment

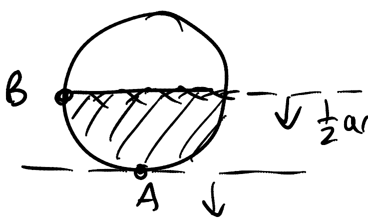
Torsion $\tau = \frac{Tc}{J} = \frac{M_z c}{J}$

Shear force $\tau = \frac{VQ}{It} \longrightarrow Q_A = 0$

Normal stress $\sigma = \frac{M_y c}{I}$



X-section of random unrelated object



$\downarrow$ $\frac{1}{2}$ area of circle below

$\downarrow$

nothing here, no area below.

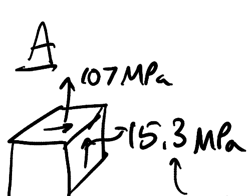
t is thickness of wood

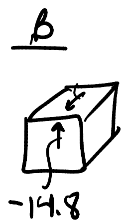
Area → (for Q) is in blue b/c we're looking at area affected by nail, from point at which you compute stress.

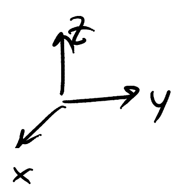
$\tau = \frac{3(.05)}{\frac{\pi}{2}(.05)^4} = 15.3$ MPa   torsion

$\tau_B = .51$
$\tau_A = 0$ Shear force

$\sigma_A = 107$ MPa
$\sigma_B = 0$ Normal stress



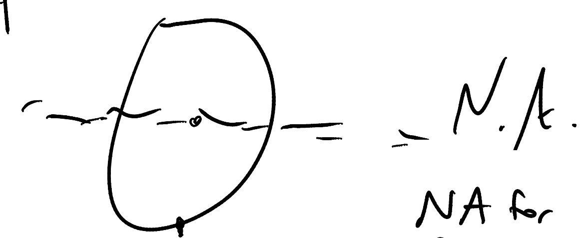




Nonsense

Tuesday, November 20, 2007
11:00 AM

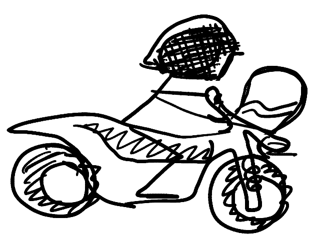
A

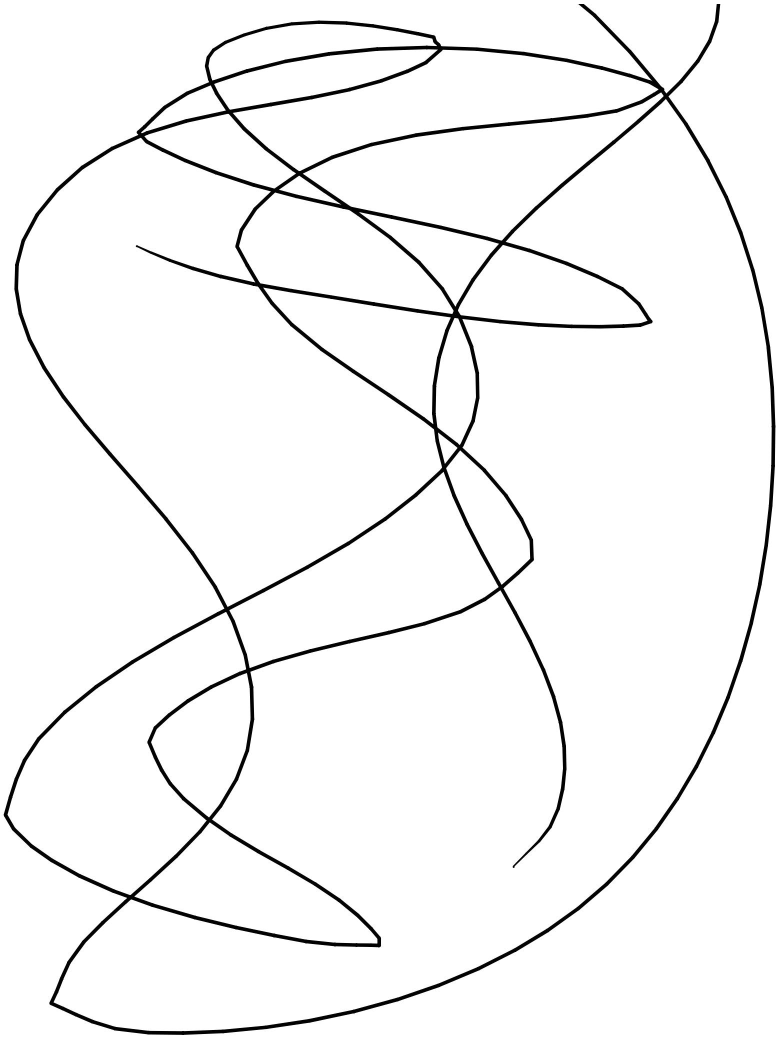


NA for circle + NA for whole object in the same place.
dist = 0

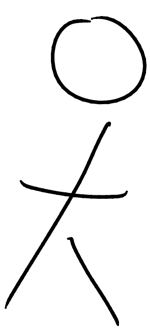
B

NA for circle + NA for half circle object in diff places.
dist = $\frac{1}{2}r$

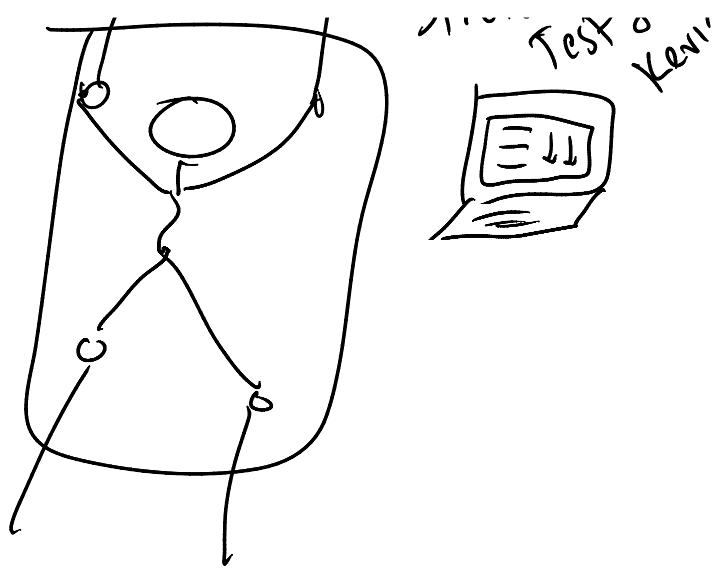




d Kevin Sue

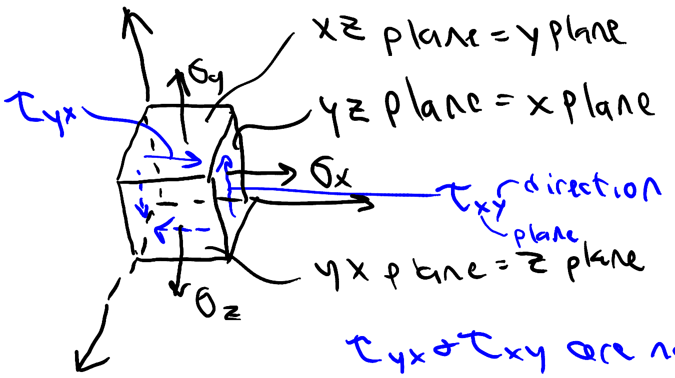


$$E_K = 30\ \overset{K}{\cancel{G}}\mathrm{Pa}.$$


Test

Plane Stress

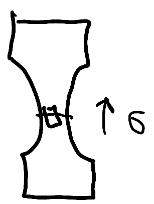
Monday, November 26, 2007
10:59 AM



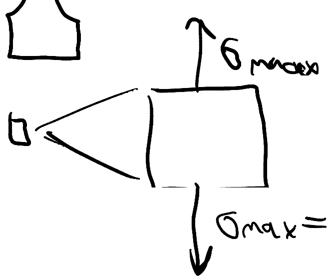
τ_{yx} & τ_{xy} are not independent

If you rotate the axes, the stresses are diff.
Need to simplify to 2D, get rid of σ_z

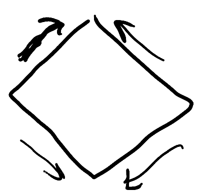
plane stress



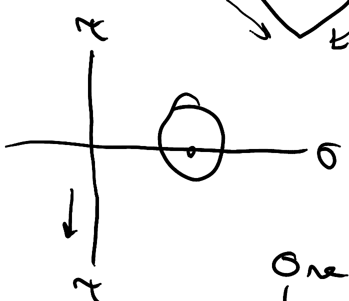
Thin samples are in plane stress
Thin walled pressure vessels have either longitudinal or hoop stress



σ_{max} = principle stress b/c no shear stress

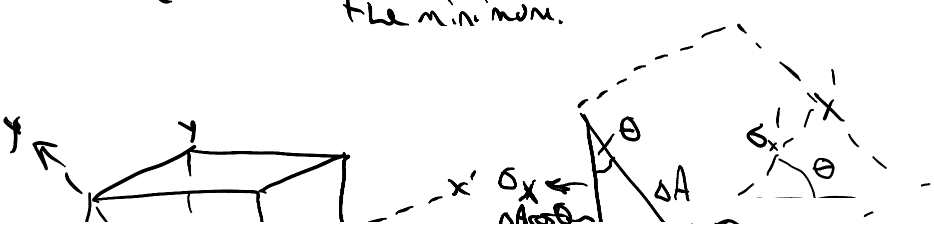


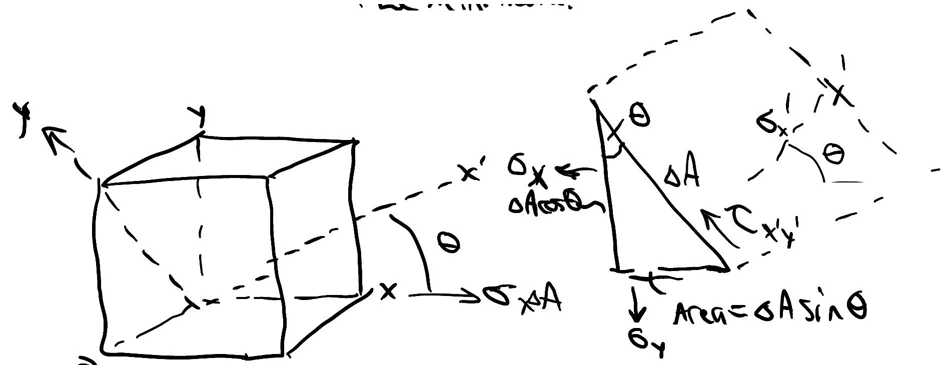
Now we have shear stress



Mohr Circle is the way to plot the stresses in the graph.

One surface has max, the other has the minimum.





$\sigma_z = 0$

ΔA is elemental area.

$$\sigma_{x'}\,\Delta A - (\underset{\text{force}}{\sigma_x \Delta A\cos\theta})\underset{\text{component}}{\cos\theta} - (\tau_{xy}\Delta A\cos\theta)\sin\theta - (\sigma_y \Delta A\sin\theta)\sin\theta - (\tau_{xy}\Delta A\sin\theta)\cos\theta = 0$$

$$\sigma_{x'} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\cos^2\theta = \frac{1+\cos\theta}{2}$$

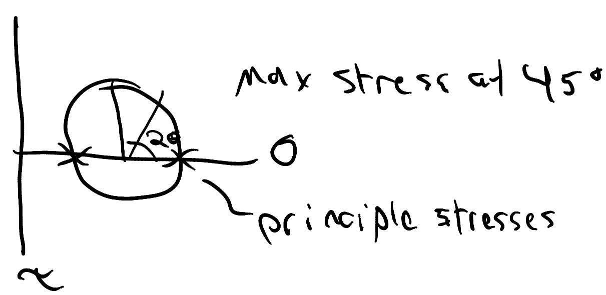
$$\sin^2\theta = \frac{1-\cos\theta}{2}$$

$$2\sin\theta\cos\theta = \sin 2\theta$$

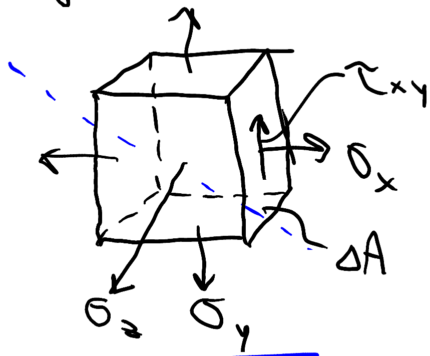
$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$$

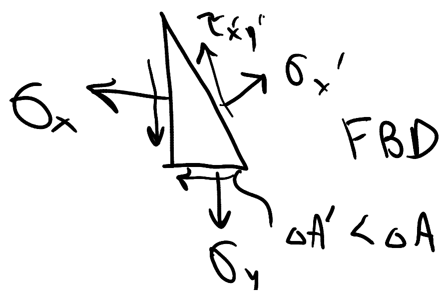


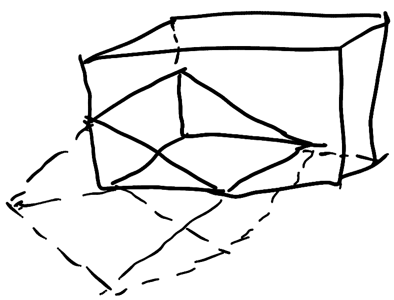
A truss is a structure intended to carry a 2 force member



6 Ind. forces
3 shear
3 normal

Take wedge out





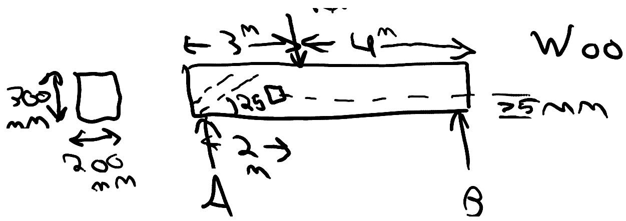
$$\sigma A = F$$

$$\Sigma F_{x'} = 0$$
$$\Sigma F_{y'} = 0$$

$$\boxed{\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y}$$

Sum of normal forces is a constant

12kN



Wood has grain = weakness

$\Sigma M_A = 0 = 12(3) + B(7)$

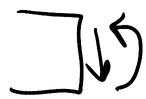
$5.143 = B$

kN

$\Sigma F = 0 = A - 12 + B$

$6.857 = A$

We need V & M now

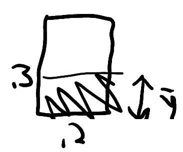


$V = 6.857$

$M = 6.857 \cdot 2 = 13.714$

$\sigma = \dfrac{My}{I} = \dfrac{(13.714)(.075\,m)}{\frac{1}{12} b h^3} = 2.2857$

$(.2)(.3)$



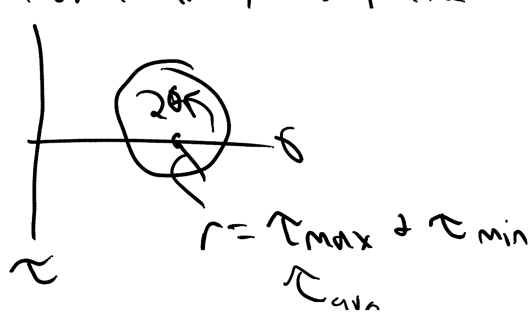
$\tau = \dfrac{VQ}{It} = \dfrac{(6.857)(A\bar{y})}{(\frac{1}{12} b h^3)(t)} = .1286$

$\sigma_{x'} = .507$

$\tau' = -.958$

$\sigma_y = 0$ b/c there is no normal stress in that direction.

Principle direction means no shear stress. One plane has max shear stress, one plane has minimum.

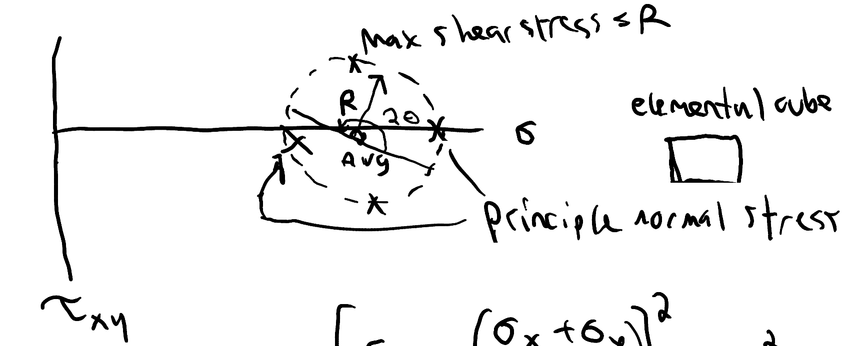


$_{avg}$

σ_{avg} = center of circle

Mohr's Circle

Plane stress - stress in the z direction is = 0



$$\left[\sigma_{x'} - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2$$

$$= \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

whole thing is constant

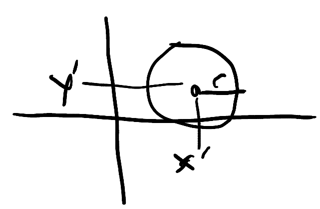
constant   const

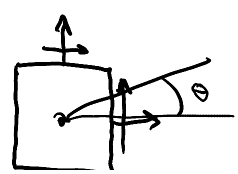
Avg normal stress as we rotate is constant.

$$R = \left(\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}\right)^{\frac{1}{2}}$$

radius

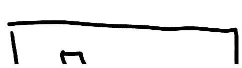
$$(\sigma_{x'}-\sigma_{Avg})^2 + \tau_{x'y'} = R^2$$
$$(x-x')^2 + (y-y')^2 = r^2$$

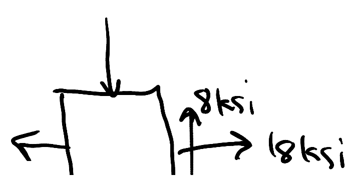




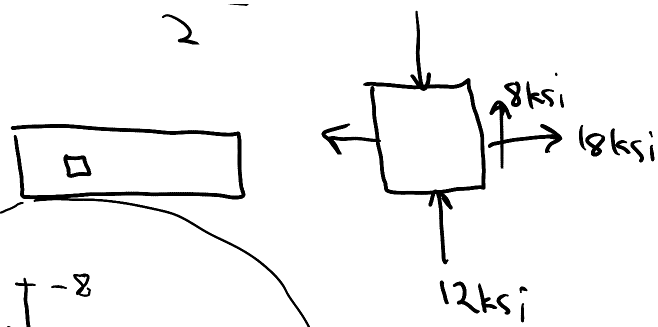
$$\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x-\sigma_y)}$$

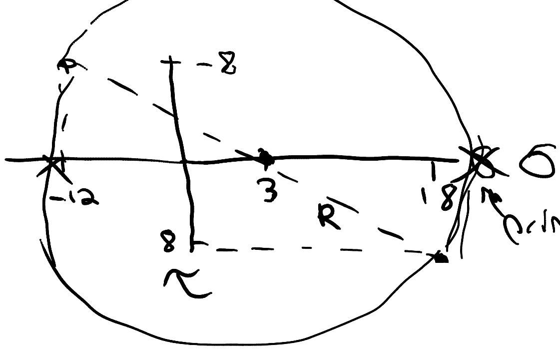
$$\sigma_x - \frac{\sigma_x+\sigma_y}{2}$$





2





Positive τ on bottom

principle normal stress

$\frac{-12+18}{2} = 3$ = Average normal stress

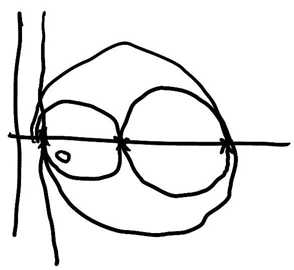
$(18-3)^2 + 8^2 = R^2$

$17 \approx R$

$17+3 = 20$ = principle normal stress, max

17 = max shear stress

There's one orientation where the normal stress is by itself - no shear stress

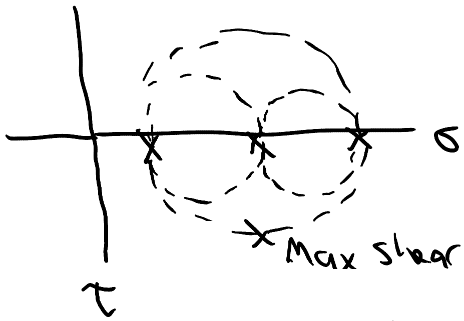


3 orthogonal axes

Maximum shear you can have is the top of the biggest circle

Deflection

Friday, November 30, 2007
10:59 AM



Each of these circles represents an axis of rotation

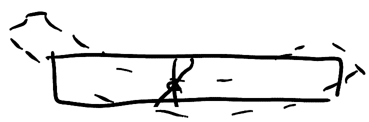
Every orientation is inside this volume

Deflection

Torsion twists

Normal stress elongates or shrinks

$\varepsilon = \frac{-y}{\rho}$



$\sigma = E\varepsilon = E\left(-\frac{y}{\rho}\right)$ $\qquad \sigma = \frac{My}{I}$

$\frac{My}{I} = E\left(-\frac{y}{\rho}\right)$ $\qquad$ Moment is constant

$$\boxed{M = \frac{EI}{\rho}}$$

$\frac{dx}{dy} =$

$\frac{dv}{dx}$ = slope as we go along in x direction

$$\frac{1}{\rho(x)} = \frac{M(x)}{EI}$$

given

$1 \quad d^2v \quad d^2v \quad M(x)$

$$\frac{1}{\rho} \approx \frac{d^2v}{dx^2} = \frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

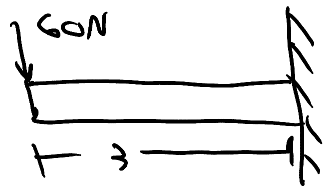
given

calculated w/ dimensions

flexural rigidity

Initial values

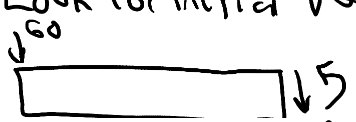
The broad concept we use is to find a boundary condition.



$E = 10\,GPa$
$I = 4e-5\ m^4$

Boundary conditions: $v=0$ when there is no deformation
$\theta = 0$ (no slope)

Look for initial v & θ. Slope & dist (deflection)



$V = -60$

$\Sigma M = 0 = 60x + M = 0$

$M(x) = -60x$

$EI\frac{d^2v}{dx^2} = -60x$

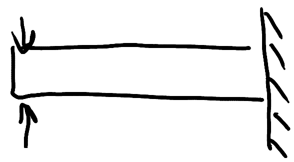
$EI\frac{dv}{dx} = -30x^2 + C_1$

$EI\,v(x) = -10x^3 + C_1x + C_2$

B.C.: $x=3$, $\frac{dv}{dx} = 0$, $v=0$

Find $C_1 = 270\ N\,m^3$
Find $C_2 = -540\ N\,m^3$

If you restrain the beam, it's an indeterminate problem.

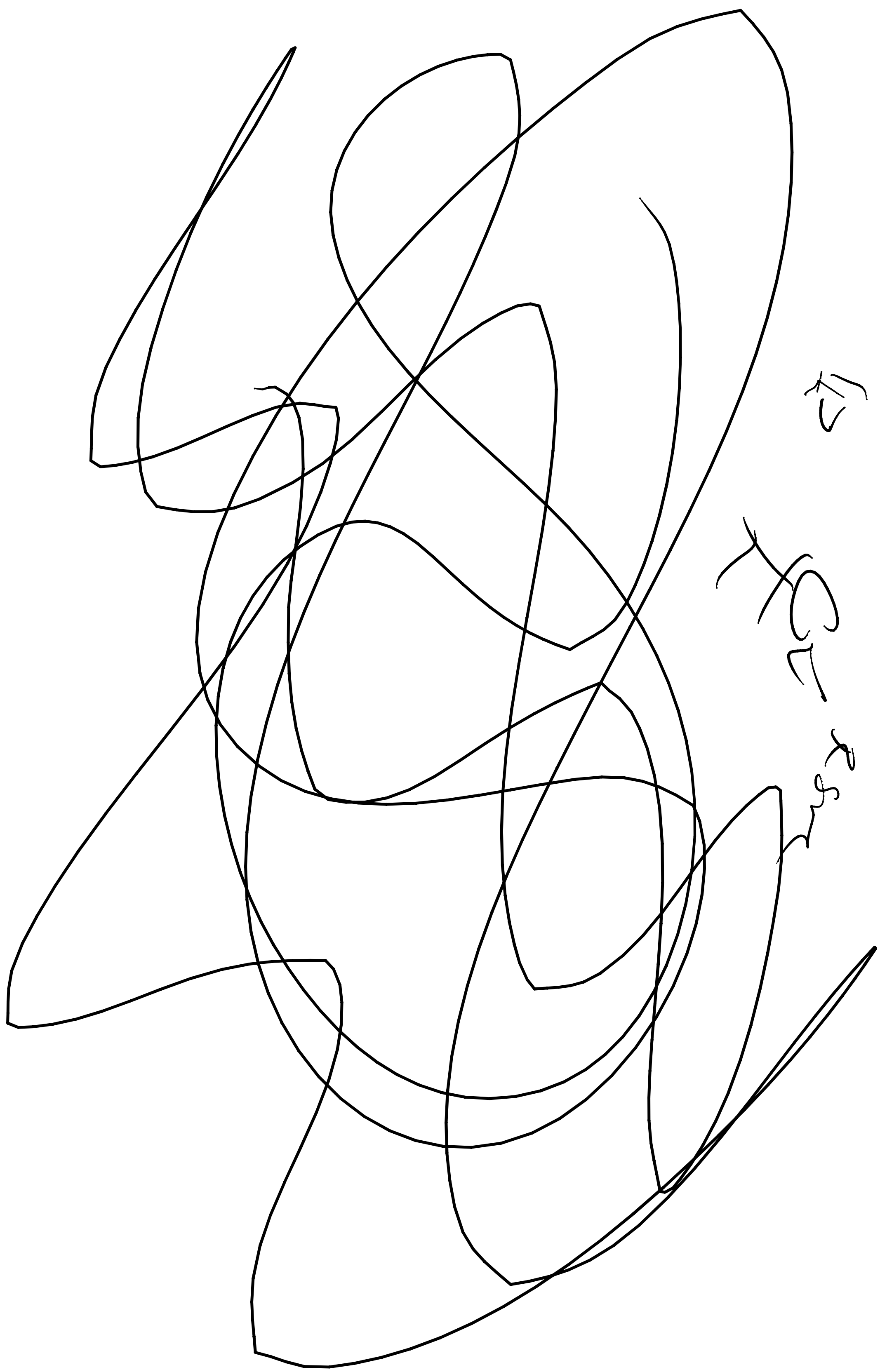


Bring material properties in
Use deflection

Take restraint away, let it deform, squash it back

Nonsense

Friday, November 30, 2007
10:59 AM

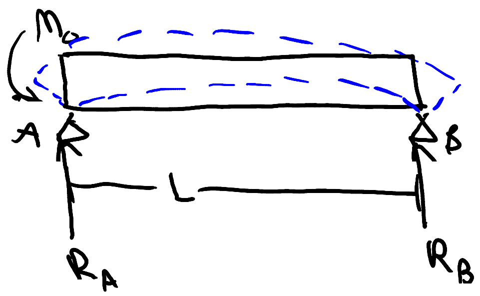


Solve for Integration Const

Monday, December 03, 2007
11:05 AM

$$EI\frac{d^2v}{dx} = M(x)$$

Solve for integration constants w/ boundary conditions



W360x64
$L = 35\,m$
$E = 200\,GPa$
$I =$

$v_A = 0$
$v_B = 0$

Max v is when $\frac{dv}{dx} = 0$

$$\Sigma M_B = 0 = -R_A L + M(x)$$

$$R_A = \frac{M_o}{L} \qquad \int \frac{M_o}{L}\,dx$$

$$\frac{M_o}{L}x + C$$

$$C = -\frac{M_o x}{L}$$

$x = L$ *

$\Rightarrow -M_o$

$$EI\frac{d^2v}{dx^2} = \frac{M_o x}{l} - M_o$$

$$EI\frac{dv}{dx} = \frac{M_o x^2}{2L} - M_o x + C_1$$

$$EIv = \frac{M_o x^3}{6L} - \frac{M_o x^2}{2} + C_1 x + C_2$$

$$\underset{x=0}{v=0} \Rightarrow C_2 = 0 \qquad \underset{x=L}{v=0} \Rightarrow C_1 = \frac{M_o L}{3}$$

$$V(x) = \frac{M_o}{ELI}\left(\frac{1}{6}x^3 - \frac{Lx^2}{2} + \frac{xL^2}{3}\right)$$

$$\frac{dv}{dx} = -\frac{1}{2}x_m^2 - Lx_m + \frac{L^2}{3} = 0$$

$x_{max} = .4232$

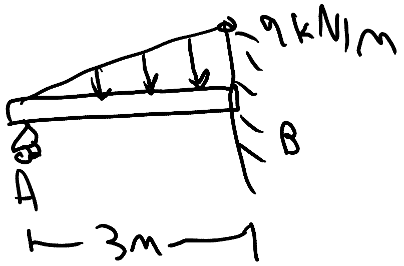
$V_{max} = 2.207E{-8}\, M_o$

if $V_{max} = 1\,m$

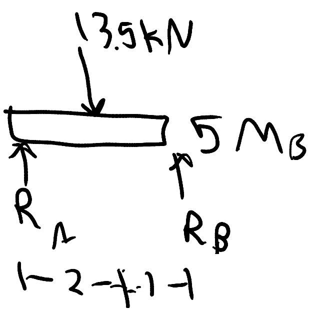
$M_o = 45.3\,kN \cdot m$

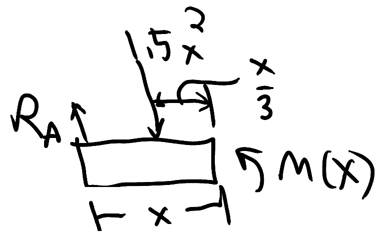
Sample prob

Tuesday, December 04, 2007
11:24 AM



$$\int w(x) = \int 3x$$





$$R_A + R_B = 13.5$$
$$3R_B + M_B - 27 = 0$$

$$\frac{dv}{dx} = 0 \text{ at } x = 3$$
$$v(0) = 0$$
$$v(3) = 0$$

$$M(x) + 1.5x^2\left(\frac{x}{3}\right) - R_A x = 0$$
$$M(x) = -.5x^3 + R_A x$$
$$EI\frac{d^2v}{dx^2} = -.5x^3 + Rx$$
$$EI\frac{dv}{dx} = -\frac{x^4}{8} + \frac{R_A}{2}x^2 + C_1$$
$$EIv = -\frac{x^5}{40} + \frac{R_A x^3}{6} + C_1 x + C_2$$

$$C_2 = 0 \qquad -\frac{3^5}{40} + \frac{3^3 R_A}{6} + 3C_1 = 0$$

$$C_1 = 2.025 - 1.5R_A$$

$$\frac{dv}{dx}(3) = 0 \quad \Rightarrow -\frac{3^4}{8} + \frac{3^2 R_A}{2} + (2.025 - 1.5R_A) = 0$$

$$\frac{dv}{dx}(3)=0 \quad \Rightarrow -\frac{3^4}{8}+\frac{3^2 R_A}{2}+(2.025-1.5R_A)=0$$

$$3R_A=8.1 \quad R_A=2.7\text{ kN}$$

$$C_1=-2.025$$

$$R_B=10.8\text{ kN}$$

$$M_B=-5.4\text{ kN}\cdot m$$

So now we have the rxn forces.
We need to find the location of max deflection.

$$EI\frac{dv}{dx}=0=-\frac{x^4}{8}+1.35x^2-2.025$$

$$EI\,v(x)=\frac{-x^5}{40}+.45x^3-2.025x$$

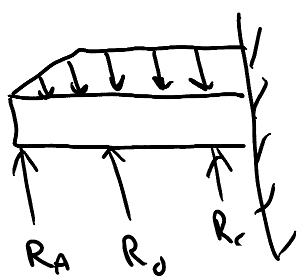
$$-\frac{x^4}{8}+1.35x^2-2.025=0 \qquad z=x_m^2$$

$$-\frac{z^2}{8}+1.35z-2.025$$

$$z=1.8 \text{ \& } 9$$

Statically indeterminant problems of the 1st degree

$$v_{max}=\frac{-1.739}{EI}$$



2nd, 3rd, 4th degree

Statically in/determinant

Wednesday, December 05, 2007
11:04 AM

Statically determinant
" indeterminant > internal, external

Statically indeterminant
- compatibility eqns, Hooke's Law
- Symmetry
- Superposition

From forces & moments, you can get - deflections
- stresses

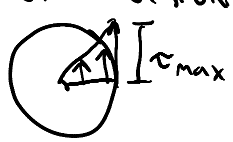
Stresses - Normal from normal force. Axial loads $\frac{F}{A} = \sigma$
- Shear from shear force. $\tau_{avg} = \frac{V}{A}$ $\tau = \frac{VQ}{It}$
- Normal from bending moments.

$$\sigma = -\frac{My}{I} \quad c = y_{max}$$

$$\sigma = \frac{Mc}{I}$$

- Shear from torsion $\tau = \frac{Tc}{J}$ or $\tau = \frac{Tr}{J}$

r = radius



- Normal from internal pressure

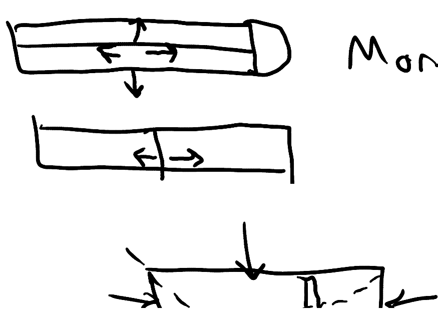
cyl tank

$$\sigma_{hoop} = \frac{p r_{inner}}{t}$$

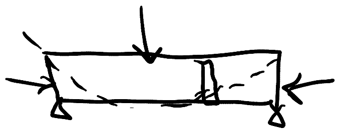
$$\sigma_{longitudinal} = \frac{p r_{inner}}{2t}$$

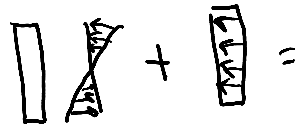
For spherical membrane,

$$\sigma_{membrane} = \sigma_{long}$$



More stress



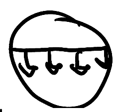


Don't need to know

- problems that involve integrating to get shear stress

  $Q = Q(y)$ ← depth of beam $\qquad \tau = \dfrac{VQ(y)}{It}$

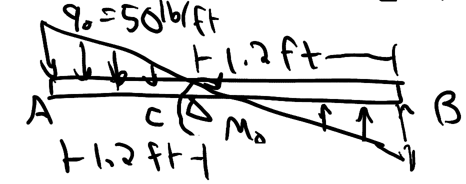
- problems that use tables (stress concentrations)
- deflections using tables (superposition)
- Matrix methods
- τ for unusual xsections.



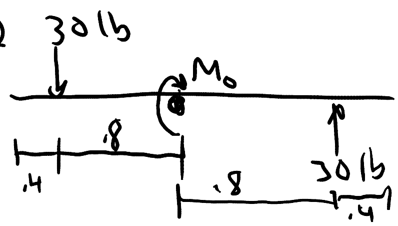
$\tau = \dfrac{4}{3}\dfrac{V}{A}$

Everything will be rectangular

Shear & Moment Diagrams



FBD



$\Sigma M_c = 0$

$M_0 = 48$ lb/ft

$C_y = 0$

$q(x) = 50 - \dfrac{50}{1.2}x = 50 - 41.667x$

$\dfrac{dv}{dx} = -q(x)$, V = shear force

$V = \int (50 - 41.667x)\,dx$

$V = -50x + 20.83x^2 + C_1$

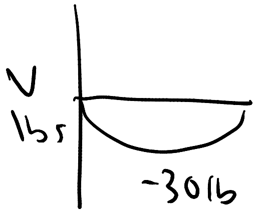
Boundary Condition

$V = 0,\ x = 0$

$\Rightarrow\ 0 = 0 + 0 + C_1$

$0 = C_1$

$V = -50x + 2.083x^2$



$M = \int V(x)\,dx$

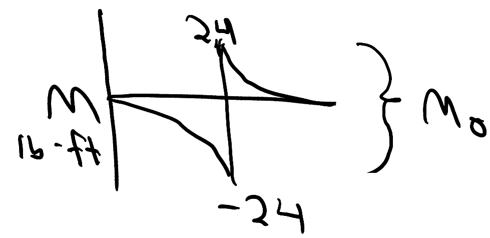
$M = -25x^2 + 6.944x^3 + C_2$

Boundary condition; $M(0) = 0$

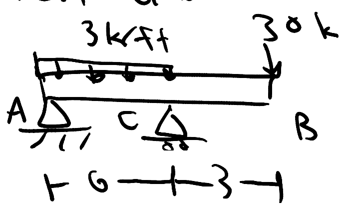
$0 = 0 + 0 + C_2$

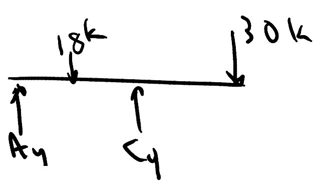
$0 = C_2$

$M = -25x + 6.944x^3$



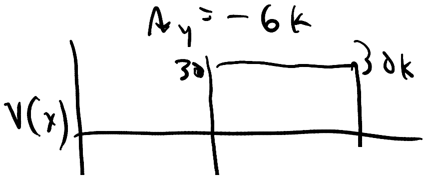
Graphical method

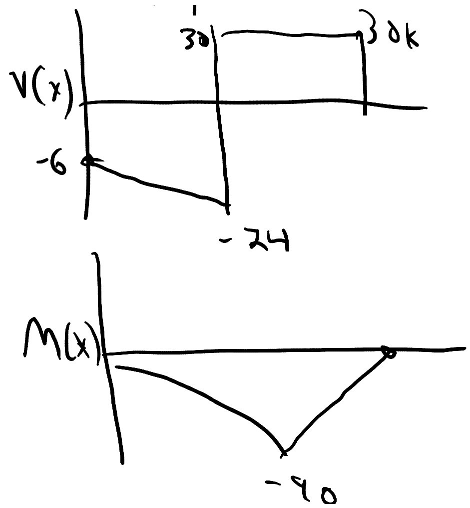




$C_y = 54\text{ k}$

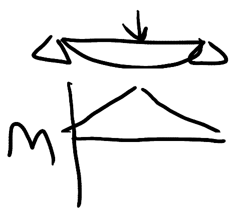
$A_y = -6\text{ k}$





Deflections

Know everything + $\boxed{M = EIv''}$ ⇒ pos. moment gives pos. curvature. (concave up)

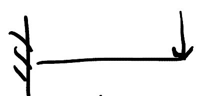


3 conditions

1. Boundary conditions

$v=0$ at $x=0$

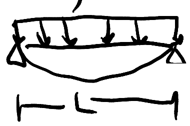
$v(L)=0$



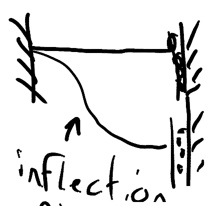
$v(0)=0$

$v'(0)=0$

2. Symmetry

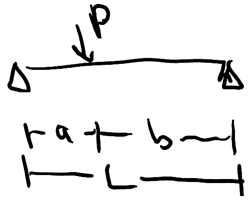


$v'(\frac{L}{2})=0$



$v''(\frac{L}{2})=0$

3. Continuity

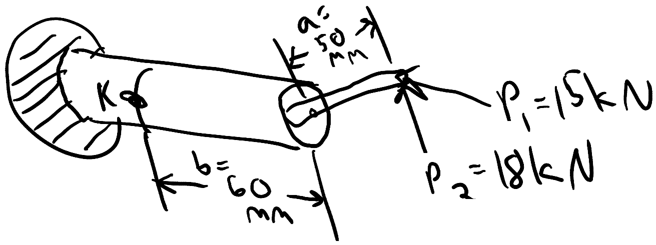


$0 \le x < a : v_1$

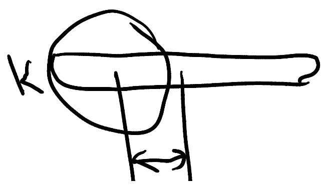
$a < x \le L : v_2$

$v_1(a) = v_2(a)$

$v_1'(a) = v_2'(a)$



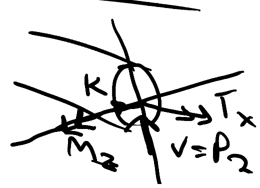
What are the stresses at K?

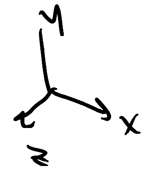


From P_1

+

From P_2





$M_z = P_2 b$

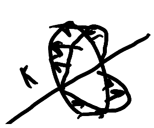
$F = P_1$

$V = P_2$

$T_x = P_2 a$

$M_y = P_1 a$

from M_z

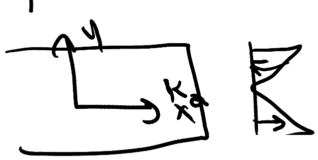


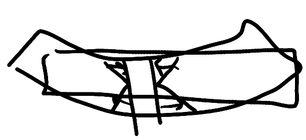
$(\sigma_x)_{M_z} = \frac{M_z r}{I_z}$ solid cylinder

$\sigma = -\frac{My}{I}$, $y = r$

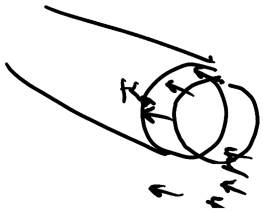
Top compression
bottom tension.

$y = 0, \ \sigma = 0$





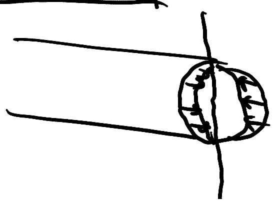
from F

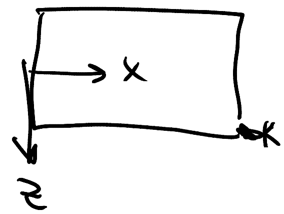


$(\sigma_x)_F = \frac{F}{A}$

$= -11.9$ MPa

from M_y

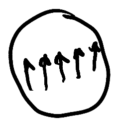


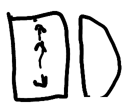


$= 119.3$ MPa

$(\sigma_x)_{M_y} = -\frac{M_y r}{I_y}$

from V





$\frac{VQ}{It} = (\tau_{xy})_V$

$\frac{4}{3}\frac{V}{A} = $ "

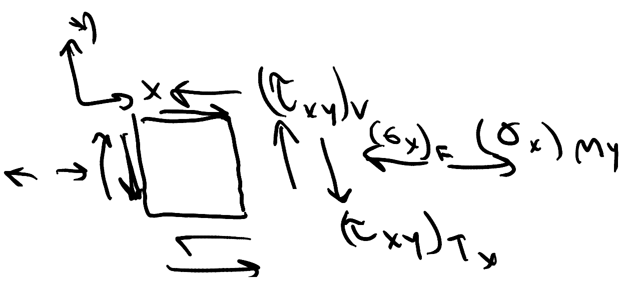
$= 19.1$ MPa

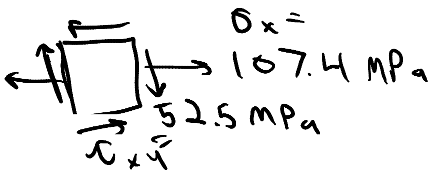
from T_x

$(\tau_{xy})_{T_x} = \frac{Tr}{J}$

$J = \frac{\pi}{2} r^4$

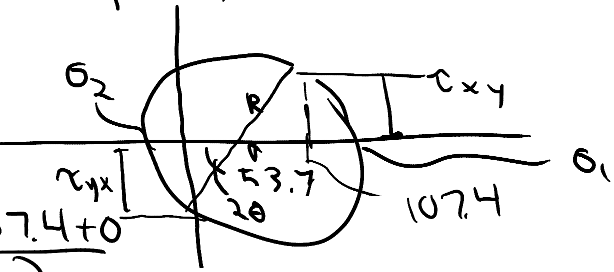
$= 71.6$ MPa





What are the maximum stresses?

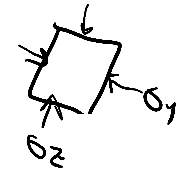
Same as principle shear & max normal



$\sigma_{Avg} = \frac{107.4+0}{2}$

$= 53.7$ MPa

Rotate by θ to get max stresses

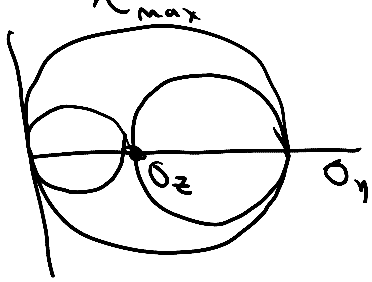


$R^2 = (53.7)^2 + (52.5)^2$

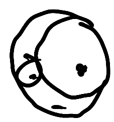
$R = 75.1$ MPa $= \tau_{max}$

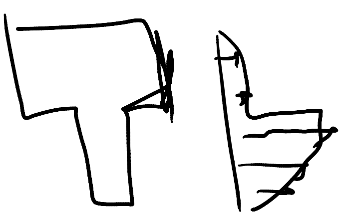
$\sigma_1 = 53.7 + R = 128.8$ MPa Max tensile stress

$\sigma_2 = -|R - 53.7| = -21.4$ MPa Max compressive stress

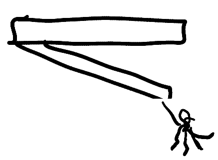


What are YOU looking at, Kevin Sue?





- Combined loads
- Statically indeterminant



Shoot the monkey

Nail spacing?

- Problem on deflection will be determinant